ALGEBRA (Q 1 & 2, PAPER 1)

1999

1 (a) Show that $\dfrac{-1+\sqrt{3}}{1+\sqrt{3}} = 2-\sqrt{3}.$

(b) Solve for x

$$\frac{4x-1}{x-3} < 2,\ x \in \mathbf{R} \text{ and } x \neq 3.$$

(c) x^2+bx+c is a factor of $x^3 - p.$

Show that

(i) $b^3 = p$

(ii) $c^3 = p^2.$

2 (a) Solve the simultaneous equations

$$x + y = 1$$
$$x^2 + y^2 = 25$$

(b) If for all integers n,

$$u_n = 2^{2n-1} + 2^{n-1},$$

show that $u_{n+1} - 2u_n - 2^{2n} = 0.$

(c) Let a, b, c be positive, unequal real numbers.

Using the results $a^2+b^2 > 2ab,\ b^2+c^2 > 2bc,\ c^2+a^2 > 2ac,$

(i) deduce that $a^2 - ab + b^2 > ab$

(ii) deduce that $a^2+b^2+c^2 > bc+ca+ab$

(iii) show that $a^3+b^3 > ab(a+b).$

ANSWERS

1 (b) $-\frac{5}{2} < x < 3$

2 (a) $(-3, 4), (4, -3)$