ALGEBRA (Q 1 & 2, PAPER 1)

1998

1 (a) Solve for x and y:

$$\frac{2x-5}{3}+\frac{y}{5}=6$$

$$\frac{3x}{10}+2=\frac{3y-5}{2}.$$

(b) If $(2x-1)$ is a factor of the polynomial

$P(x)=2x^3-5x^2-kx+3,$

find the value of k.

Find the other two factors of $P(x)$.

(c) If the quadratic equation $ax^2+bx+c=0$ has equal roots, solve for x in terms of a and b, where $a,b,c\in\mathbf{R}$.

By letting $x=3^y$, write

$t3^y+3^{-y}=3$

as a quadratic equation in x, where $t\in\mathbf{R}$ and $t\neq 0$.

Find the value of t for which this equation has equal roots.

Assuming this value of t, solve the equation

$t3^y+3^{-y}=3.$

2 (a) Solve for x: $|x-4|<5$.

(b) If α and β are the roots of the equation

$x^2-6x+2=0,\ \alpha>0,\ \beta>0,$

find $\alpha\beta$ and $\alpha+\beta$.

Factorise $\alpha^3+\beta^3$.

Find the value of $\alpha^3+\beta^3$.

(c) Show for all real numbers a, $b > 0$, that

$a+b\geq 2\sqrt{ab}.$

Show that $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{(a+b)^2}{ab}$.

Deduce that $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)\geq 4$.

ANSWERS

1 (a) $x = 10$, $y = 5$

(b) $k = 4$; $(x-3)(x+1)$

(c) $x = -\frac{b}{2a}$; $tx^2 - 3x + 1 = 0$; $\frac{9}{4}$; $\log_3(\frac{2}{3})$

2 (a) $-1 < x < 9$

(b) $\alpha\beta = 2$; $\alpha + \beta = 6$; $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$; 180