ALGEBRA (Q 1 & 2, PAPER 1)

1997

1 (a) If $x=\sqrt{a}+\frac{1}{\sqrt{a}}$ and $y=\sqrt{a}-\frac{1}{\sqrt{a}}$, $a>0$, find the value of $\sqrt{x^2-y^2}$.

(b) Let $f(x)=ax^3+bx^2+cx+d$, where $a,b,c,d\in\mathbf{R}$. If k is a real number such that $f(k)=0$, prove that $x-k$ is a factor of $f(x)$.

(c) If $(x-1)^2$ is a factor of ax^3+bx^2+1, find the value of a and the value of b.

2 (a) Solve the simultaneous equations

$$2x-3y=1$$
$$x^2+xy-4y^2=2$$

(b) Solve

$$x^2-6x+8=0$$

and hence find the values of x for which

$$(x+\tfrac{1}{x})^2-6(x+\tfrac{1}{x})+8=0, x\in\mathbf{R} \text{ and } x\neq 0.$$

(c) Let $f(x)=\frac{1}{x}$ for all $x\in\mathbf{R}$ and $x\neq 0$.
Points a and b have coordinates $(p, f(p))$ and $(q, f(q))$, respectively, for $0<p<q$.

(i) Show that the equation of the line ab can be written as

$$y=g(x)=\tfrac{1}{p}-\tfrac{1}{pq}(x-p).$$

(ii) Show that

$$f(x)-g(x)=\frac{(x-q)(x-p)}{pqx}.$$

Hence, show that $f(x)-g(x)<0$ for $0<p<x<q$.

ANSWERS

1 (a) 2

(c) $a=2, b=-3$

2 (a) (2, 1), (11, 7)

(b) $x=2,4; x=1, 2\pm\sqrt{3}$