ALGEBRA (Q 1 & 2, PAPER 1)

1996

1 (a) Express $\frac{1-\sqrt{2}}{1+\sqrt{2}}$ in the form $a\sqrt{2}-b$, where $a, b \in \mathbf{N}$.

(b) (i) $(x + 1)$ is a factor of $x^3+5x^2+kx-12$.
Find the value of k and the other two factors of the cubic expression.

(ii) If $x=\sqrt{p}+\frac{1}{\sqrt{p}}+1$ where $p > 0$, express x^2-2x in terms of p.

(c) (i) Make a sketch of the region of the plane represented by
$y \geq |x|$ and $y \leq 2+|x|$.

(ii) x^2-px+1 is a factor of ax^3+bx+c where $a \neq 0$.
Show $c^2 = a(a-b)$.

2 (a) Solve for x, y and z

$$x+y-z=0$$
$$x-y+z=4$$
$$x-y-z=-8$$

(b) (i) Solve for x

$$\frac{2x-7}{x+3}<1,\ x \neq 3.$$

(ii) If $u_n = n!(n+2)$ show that
$(n+1)u_n+(n+1)!=u_{n+1}$.

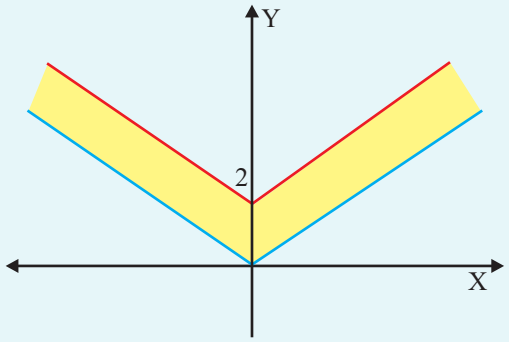
(c) Find the quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ given that $\alpha+\beta=5$ and $\alpha\beta=k$, where $k \neq 0$.
Find the range of values of k for which the equation will have real roots.

ANSWERS

1 (a) $2\sqrt{2}-3$

(b) (i) $k=-8,\ (x+6)(x-2)$ (ii) $p+\frac{1}{p}+1$

(c) (i)



2 (a) $x = 2, y = 4, z = 6$

(b) (i) $-3 < x < 10$

(c) $kx^2-5x+1=0,\ k \le \frac{25}{4}$