

**TRIGONOMETRY (Q 5, PAPER 2)****LESSON NO. 7: COMPOUND ANGLES****2001**

5 (a)  $\sin \theta = \frac{3}{5}$  where  $0^\circ < \theta < 90^\circ$ .

Find, without using the Tables or a calculator, the value of

(i)  $\cos \theta$

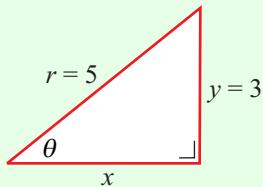
(ii)  $\cos 2\theta$ . [Note:  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ .]

**SOLUTION****5 (a) (i)**

Draw a right-angled triangle.

$$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \dots\dots \quad 4$$

$$\sin \theta = \frac{3}{5} = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$



Use Pythagoras to find x.

$$x^2 + y^2 = r^2 \quad \dots\dots \quad 1$$

$$x^2 + 3^2 = 5^2 \Rightarrow x^2 + 9 = 25$$

$$\Rightarrow x^2 = 16$$

$$\therefore x = \sqrt{16} = 4$$

$$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \dots\dots \quad 3$$

$$\therefore \cos \theta = \frac{4}{5}$$

**5 (a) (ii)**

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\Rightarrow \cos 2\theta = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25}$$

$$\therefore \cos 2\theta = \frac{7}{25}$$

**1997**

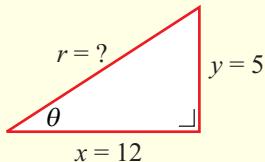
- 5 (b)  $\theta$  is an acute angle where  $\tan \theta = \frac{5}{12}$ .

Find, as a fraction, the value of

- $\cos \theta$
- $\sin \theta$
- $\cos 2\theta$ . [Note:  $\cos 2\theta = \cos(\theta + \theta)$ .]

**SOLUTION**

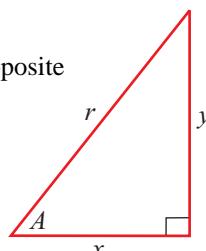
$$\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}} \quad \dots \dots \quad 5$$

**PYTHAGORAS**

One of the angles in a right-angled triangle is  $90^\circ$ . The side opposite this angle is called the **hypotenuse**.

Pythagoras' theorem applies to right-angled triangles.

$$x^2 + y^2 = r^2 \quad \dots \dots \quad 1$$

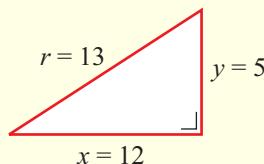


$$x^2 + y^2 = r^2 \Rightarrow 12^2 + 5^2 = r^2$$

$$\Rightarrow 144 + 25 = r^2$$

$$\Rightarrow r^2 = 169$$

$$\therefore r = \sqrt{169} = 13$$

**5 (b) (i)**

$$\cos \theta = \frac{12}{13}$$

$$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \dots \dots \quad 3$$

**5 (b) (ii)**

$$\sin \theta = \frac{5}{13}$$

$$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \dots \dots \quad 4$$

**5 (b) (iii)**

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \cos 2\theta = \frac{144}{169} - \frac{25}{169}$$

$$\therefore \cos 2\theta = \frac{119}{169}$$

**1996**

- 5 (b)  $A$  and  $B$  are acute angles where  $\sin A = \frac{3}{5}$  and  $\cos B = \frac{5}{13}$ .

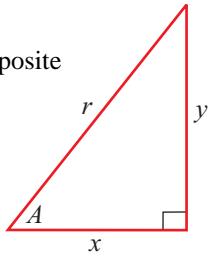
Find, as fractions, the value of  $\cos A$  and the value of  $\sin B$ .

Find the value of  $\sin(A + B)$ , giving your answer as a single fraction.

**SOLUTION****PYTHAGORAS**

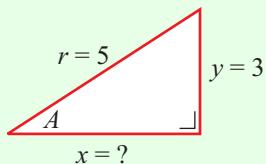
One of the angles in a right-angled triangle is  $90^\circ$ . The side opposite this angle is called the **hypotenuse**.

Pythagoras' theorem applies to right-angled triangles.



$$x^2 + y^2 = r^2 \quad \dots \dots \quad 1$$

$$\sin A = \frac{3}{5}$$



$$x^2 + y^2 = r^2 \Rightarrow x^2 + 3^2 = 5^2$$

$$\Rightarrow x^2 + 9 = 25$$

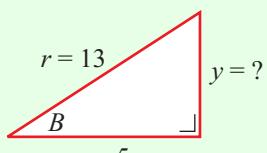
$$\Rightarrow x^2 = 16$$

$$\therefore x = \sqrt{16} = 4$$

$$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \dots \dots \quad 4$$

$$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \dots \dots \quad 3$$

$$\cos B = \frac{5}{13}$$



$$x^2 + y^2 = r^2 \Rightarrow 5^2 + y^2 = 13^2$$

$$\Rightarrow 25 + y^2 = 169$$

$$\Rightarrow y^2 = 144$$

$$\therefore y = \sqrt{144} = 12$$

$$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \dots \dots \quad 4$$

$$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \dots \dots \quad 3$$

$$\therefore \sin B = \frac{12}{13}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\Rightarrow \sin(A + B) = \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$\Rightarrow \sin(A + B) = \frac{15}{65} + \frac{48}{65}$$

$$\therefore \sin(A + B) = \frac{63}{65}$$

**COMPOUND ANGLES**

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$