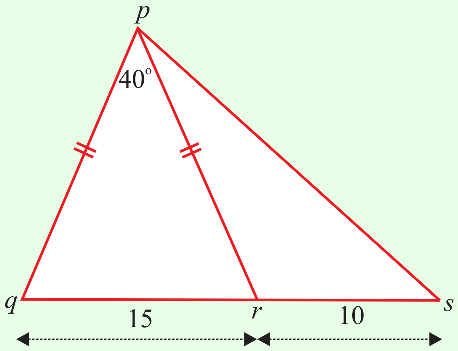
TIIGONOMETRY (Q 5, PAPER 2)

LESSON NO. 6: MORE DIFFICULT TRIANGLES

2007

5 (c) In the triangle pqr,
$|pq| = |pr|$, $|qr| = 15$ cm and $|rpq| = 40°$.
(i) Find $|pr|$, correct to the nearest centimetre.

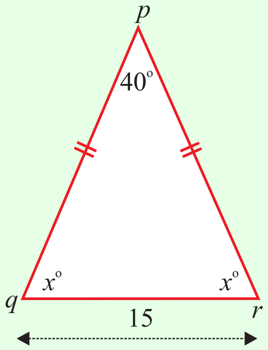
(ii) s is a point on qr such that $|rs| = 10$ cm.
Find $|ps|$, correct to the nearest centimetre.



SOLUTION

5 (c) (i)

Triangle pqr is an isosceles triangle. Therefore, the angles opposite the equal sides are also equal. Call these angles $x°$. The three angles in a triangle add up to 180°.



$$\therefore x^\circ + x^\circ + 40^\circ = 180^\circ$$

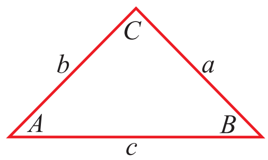
$$\Rightarrow 2x^\circ = 140^\circ$$

$$\therefore x^\circ = 70^\circ$$

SINE RULE FORMULA

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$ .... 9 *OR* $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ 9

You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



REMEMBER IT AS:

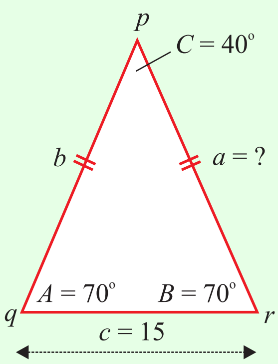
$$\frac{\sin(\text{Angle 1})}{\text{Opposite side}} = \frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad OR \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})} = \frac{\text{Opposite side}}{\sin(\text{Angle 2})}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 70^\circ} = \frac{15}{\sin 40^\circ}$$

$$\Rightarrow a = \frac{15 \sin 70^\circ}{\sin 40^\circ}$$

$$\therefore a = |pr| = 22 \text{ cm}$$

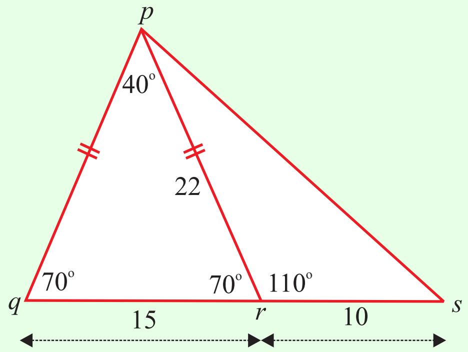


CONT....

5 (c) (ii)

$70^\circ + |\angle prs| = 180^\circ$

$\therefore |\angle prs| = 180^\circ - 70^\circ = 110^\circ$



The Cosine Rule

$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ **10**

You use the Cosine rule when you are given:
[A] Two sides and one included angle,
[B] Three sides.

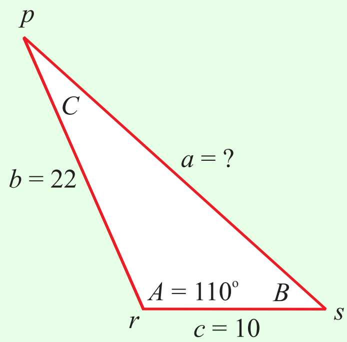
There are two other versions of the cosine rule not given in the tables:

$$\boxed{\begin{array}{l} b^2 = a^2 + c^2 - 2ac\cos B \\ c^2 = a^2 + b^2 - 2ab\cos C \end{array}}$$ **10**

$a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow a^2 = 22^2 + 10^2 - 2(22)(10)\cos 110^\circ$

$\therefore a = |ps| = 27$ cm

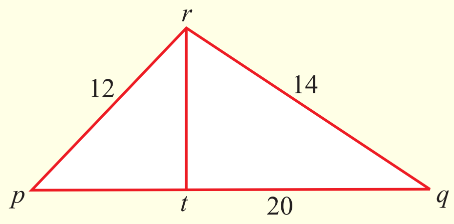


2006

5 (c) The lengths of the sides of the triangle pqr are $|pq| = 20$, $|qr| = 14$ and $|pr| = 12$.

(i) Find $|\angle rpq|$, correct to one decimal place.

(ii) Find $|rt|$, where $rt \perp pq$. Give your answer correct to the nearest whole number.



SOLUTION

5 (c) (i)

THE COSINE RULE

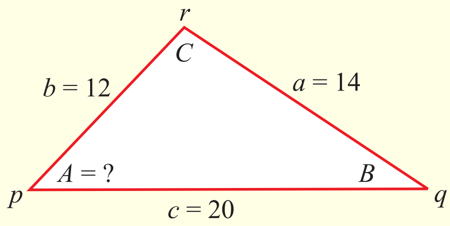
$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ 10

You use the Cosine rule when you are given:
[A] Two sides and one included angle,
[B] Three sides.

There are two other versions of the cosine rule not given in the tables:

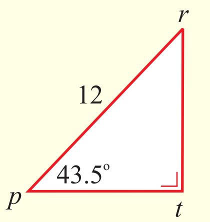
$$\boxed{\begin{aligned} b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$ 10

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Rightarrow 14^2 = 12^2 + 20^2 - 2(12)(20)\cos A$$
$$\Rightarrow 196 = 144 + 400 - 480\cos A$$
$$\Rightarrow 480\cos A = 144 + 400 - 196$$
$$\Rightarrow 480\cos A = 348$$
$$\Rightarrow \cos A = \frac{348}{480}$$
$$\therefore A = |\angle rpq| = \cos^{-1}\left(\frac{348}{480}\right) = 43.5^\circ$$



5 (c) (ii)

$\boxed{\sin A = \dfrac{y}{r} = \dfrac{\text{Opposite}}{\text{Hypotenuse}}}$ 4



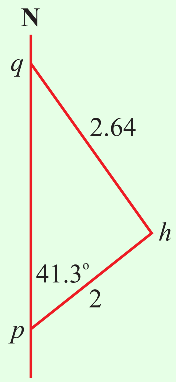
$$\sin 43.5^\circ = \frac{|rt|}{12} \Rightarrow |rt| = 12\sin 43.5^\circ$$
$$\therefore |rt| = 8$$

2005

5 (c) A lighthouse, h, is observed from a ship sailing a straight course due North.
The distance from p to h is 2 km and the bearing of the lighthouse from p is N 41.3° E.
The distance from q to h is 2.64 km.

(i) Find the bearing of the lighthouse from q.

(ii) The ship is sailing at a speed of 19 km/h. Find, correct to the nearest minute, the time taken to sail from p to q.



SOLUTION

5 (c) (i)

SINE RULE FORMULA

$$\boxed{\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}} \quad \text{.......} 9 \qquad \textit{OR} \qquad \boxed{\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}} \quad \text{.......} 9$$

You use the Sine Rule when you are given:
[A] Two angles and one side.
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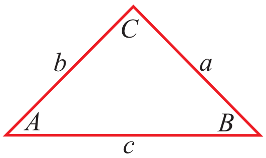


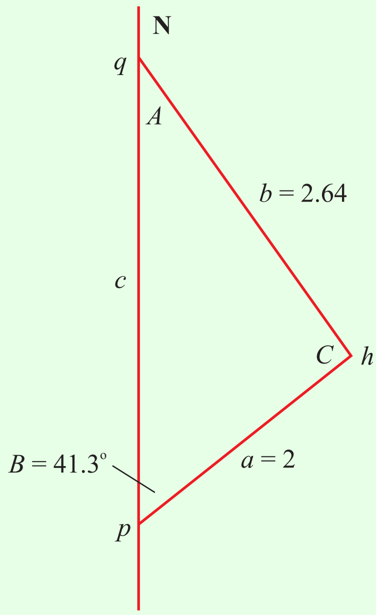
REMEMBER IT AS:

$$\boxed{\frac{\sin(\text{Angle 1})}{\text{Opposite side}}=\frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad \textit{OR} \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})}=\frac{\text{Opposite side}}{\sin(\text{Angle 2})}}$$

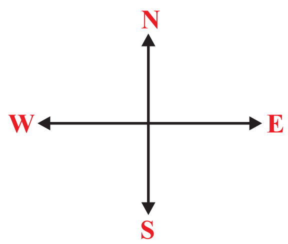
$$\frac{\sin A}{a}=\frac{\sin B}{b} \Rightarrow \frac{\sin A}{2}=\frac{\sin 41.3^\circ}{2.64}$$

$$\Rightarrow \sin A=\frac{2\sin 41.3^\circ}{2.64}=0.5$$

$$\therefore A=\sin^{-1}(0.5)=30^\circ$$

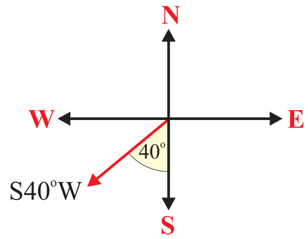


COMPASS DIRECTIONS



The compass directions are shown. The four main directions are North (**N**), South (**S**), East (**E**) and West (**W**). Other directions are combinations of these.

Ex. S40°W means start at a point on the Southern direction and go 40° towards the West.



The bearing of the lighthouse from q is S 30° E.

CONT....

5 (c) (ii)

Find the angle C. The 3 angles of a triangle add up to 180°.

$C + 41.3^\circ + 30^\circ = 180^\circ \Rightarrow C = 108.7^\circ$

Use the Sine Rule to find the distance $|qp|$.

$$\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow \frac{c}{\sin 108.7^\circ} = \frac{2}{\sin 30^\circ}$$

$$\Rightarrow c = \frac{2\sin 108.7^\circ}{\sin 30^\circ}$$

$$\therefore c = |qp| = 3.8 \text{ km}$$

$$\text{Speed } (v) = \frac{\text{Distance } (s)}{\text{Time } (t)} \qquad \boxed{v = \frac{s}{t}} \; \ldots\ldots\; 4$$

$v = 19$ km/h

$s = 3.8$ km

$t = ?$

$$v = \frac{s}{t} \Rightarrow 19 = \frac{3.8}{t}$$

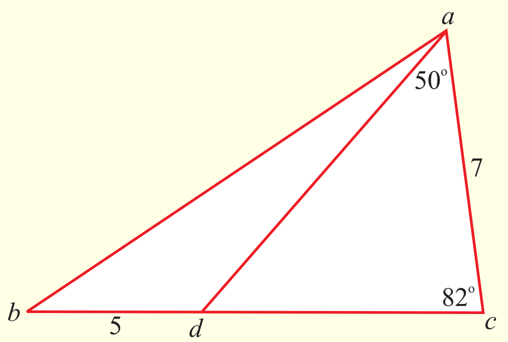
$$\Rightarrow t = \frac{3.8}{19} = 0.2 \text{ h} = 12 \text{ minutes}$$

2004

5 (c) In the triangle abc, d is a point on $[bc]$.
$|bd| = 5$ cm, $|ac| = 7$ cm,
$|\angle dca| = 82^\circ$ and $|\angle cad| = 50^\circ$.

(i) Find $|dc|$, correct to the nearest cm.

(ii) Find $|ab|$, correct to the nearest cm.



SOLUTION

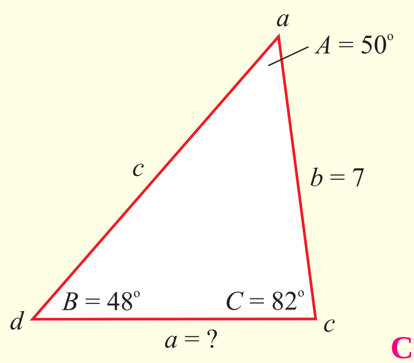
5 (c) (i)

Consider the triangle adc.
The 3 angles add up to 180° so you can calculate angle B.

$B + 50^\circ + 82^\circ = 180^\circ$

$\Rightarrow B = 180^\circ - 50^\circ - 82^\circ$

$\therefore B = 48^\circ$



CONT....

SINE RULE FORMULA

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \ldots\ldots\ \textbf{9} \qquad OR \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \ldots\ldots\ \textbf{9}$$

You use the Sine Rule when you are given:
[A] Two angles and one side.
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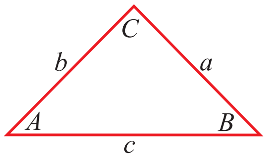

REMEMBER IT AS:

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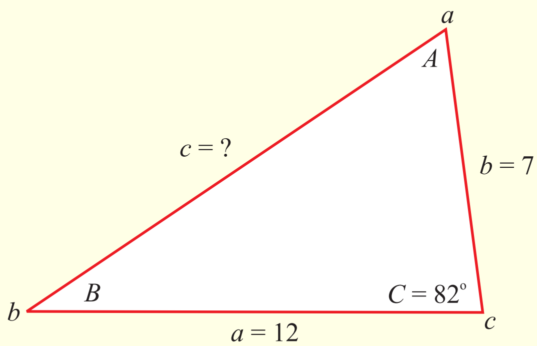
$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{a}{\sin 50^\circ} = \frac{7}{\sin 48^\circ}$$

$$\Rightarrow a = |dc| = \frac{7 \sin 50^\circ}{\sin 48^\circ} = 7 \text{ cm}$$

5 (c) (ii)

Consider the triangle *abc*.



THE COSINE RULE

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \ldots\ldots\ \textbf{10}$$

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There are two other versions of the cosine rule not given in the tables:

$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C \quad \ldots\ldots\ \textbf{10}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\Rightarrow c^2 = 12^2 + 7^2 - 2(12)(7) \cos 82^\circ$$

$$\Rightarrow c^2 = 144 + 49 - 168 \cos 82^\circ$$

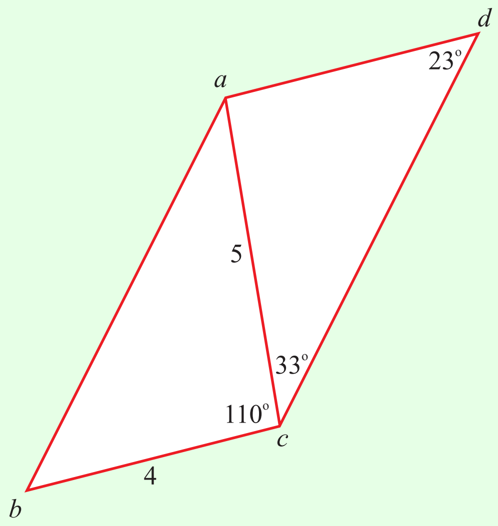
$$\therefore c = |ab| = 13 \text{ cm}$$

2002

5 (c) In the quadrilateral *abcd*, $|ac| = 5$ units, $|bc| = 4$ units, $|\angle bca| = 110^\circ$, $|\angle acd| = 33^\circ$ and $|\angle cda| = 23^\circ$.

(i) Calculate $|ab|$, correct to two decimal places.

(ii) Calculate $|cd|$, correct to two decimal places.



SOLUTION

5 (c) (i)

Consider triangle *abc*.

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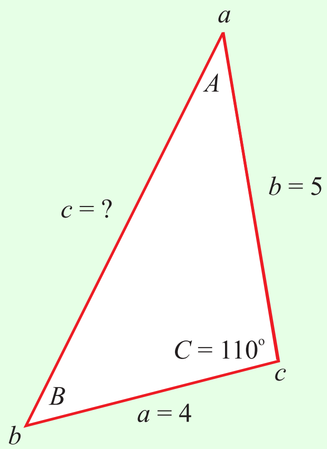
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
.... **10**

$c^2 = a^2 + b^2 - 2ab\cos C$

$\Rightarrow c^2 = 4^2 + 5^2 - 2(4)(5)\cos 110^\circ$

$\Rightarrow c^2 = 16 + 25 - 40\cos 110^\circ$

$\therefore c = |ab| = 7.39$ cm

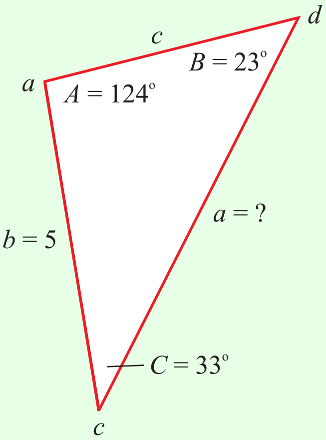


5 (c) (ii)

Consider triangle *adc*.
The 3 angles in a triangle add up to 180°.

$A + 33^\circ + 23^\circ = 180^\circ$

$\therefore A = 124^\circ$



CONT....

Sine Rule Formula

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \text{ .... } \mathbf{9} \quad OR \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ .... } \mathbf{9}$$

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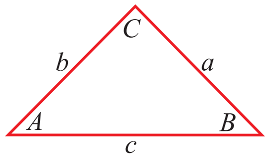

Remember it as:

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$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{a}{\sin 124^\circ} = \frac{5}{\sin 23^\circ}$$

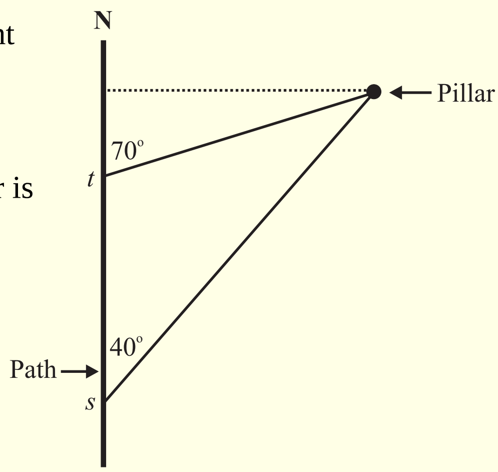
$$\Rightarrow a = \frac{5 \sin 124^\circ}{\sin 23^\circ}$$

$$\therefore a = |cd| = 10.61 \text{ cm}$$

2001

5 (c) s and t are two points 300 m apart on a straight path due north.
From s the bearing of a pillar is N40°E.
From t the bearing of the pillar is N70°E.
(i) Show that the distance from t to the pillar is 386 m, correct to the nearest metre.

(ii) Find the shortest distance from the path to the pillar, correct to the nearest metre.



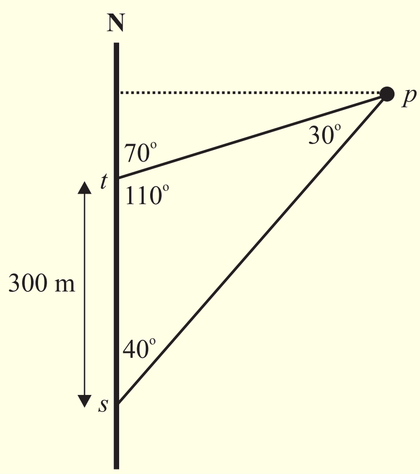
Solution

5 (c) (i)

Call the pillar p.

$|\angle pts| = 110^\circ$ [Straight angle]

$|\angle tps| = 30^\circ$ [3 angles of a triangle add up to 180°]



Cont....

Consider triangle *pts*:

SINE RULE FORMULA

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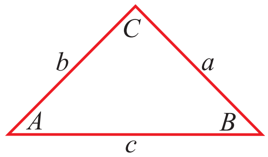

REMEMBER IT AS:

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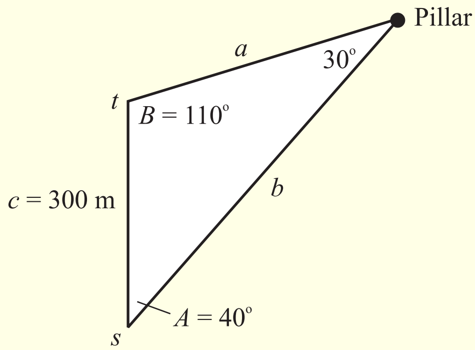
$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{a}{\sin 40^\circ} = \frac{300}{\sin 30^\circ}$$

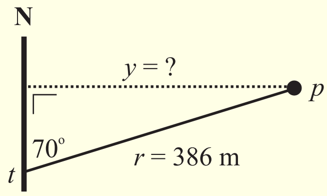
$$\Rightarrow a = \frac{300 \sin 40^\circ}{\sin 30^\circ}$$

$$\therefore a = |pt| = 386 \text{ m}$$



5 (c) (ii)

$$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$ 4



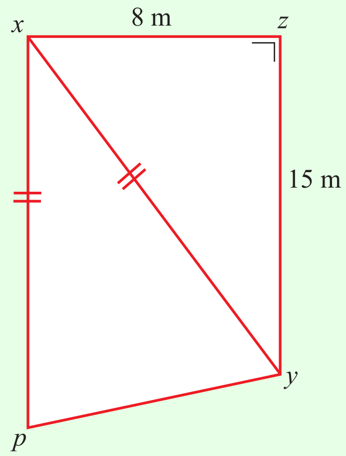
$$\sin 70^\circ = \frac{y}{386} \Rightarrow y = 386 \sin 70^\circ$$

$\therefore$ Shortest distance $y = 363$ m

2000

5 (c) (i) In the diagram, the triangle zxy is right-angled.
$|zx| = 8$ m and $|zy| = 15$ m.
Find $|xy|$.

(ii) xp is parallel to zy.
$|xp| = |xy|$, as shown.
Calculate $|py|$, correct to the nearest metre.

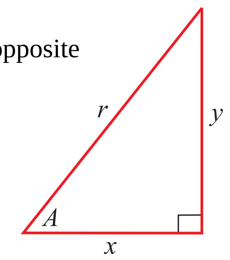


SOLUTION

5 (c) (i)

PYTHAGORAS
One of the angles in a right-angled triangle is 90°. The side opposite this angle is called the **hypotenuse**.
Pythagoras' theorem applies to right-angled triangles.

$\boxed{x^2 + y^2 = r^2}$ 1



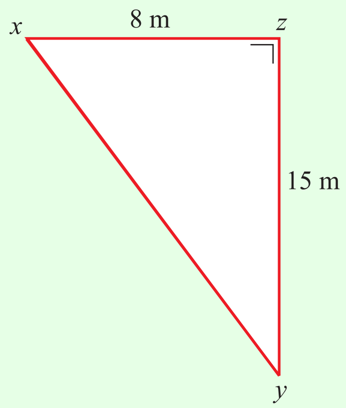
$|xz|^2 + |yz|^2 = |xy|^2$

$\Rightarrow 8^2 + 15^2 = |xy|^2$

$\Rightarrow 64 + 225 = |xy|^2$

$\Rightarrow 289 = |xy|^2$

$\therefore |xy| = \sqrt{289} = 17$ m



5 (c) (ii)

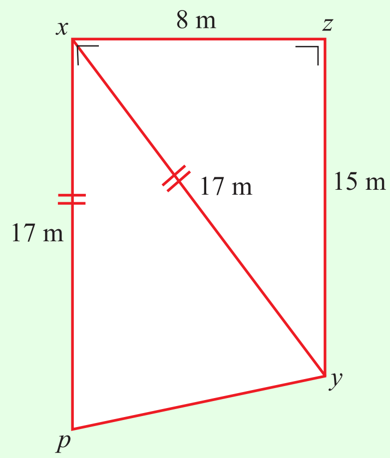
$|\angle zxp| = 90^\circ$ [zy is parallel to xp]

$\boxed{\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}}$ 5

$\tan|\angle zxy| = \frac{15}{8} \Rightarrow |\angle zxy| = \tan^{-1}(\frac{15}{8})$

$\therefore |\angle zxy| = 61.9^\circ$

$\Rightarrow |\angle yxp| = 90^\circ - 61.9^\circ = 28.1^\circ$



CONT....

THE COSINE RULE

$$a^2 = b^2 + c^2 - 2bc\cos A$$ **10**

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[A] Two sides and one included angle,
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There are two other versions of the cosine rule not given in the tables:

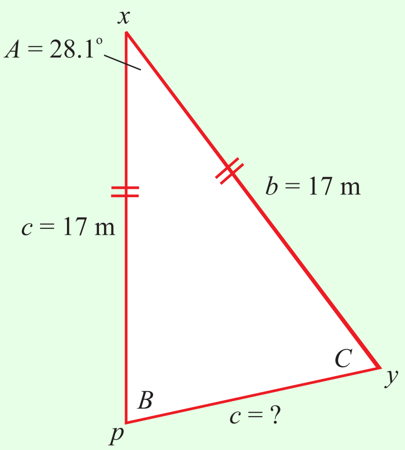
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$ **10**

$a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow a^2 = 17^2 + 17^2 - 2(17)(17)\cos 28.1^\circ$

$\Rightarrow a^2 = 289 + 289 - 578\cos 28.1^\circ$

$\therefore a = |py| = 8$ m



1998

5 (c) Three ships are situated in a straight line at points a, b and c.
p is a port such that

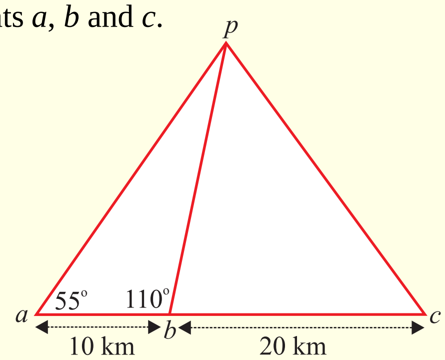
$|\angle bap| = 55^\circ$, $|\angle abp| = 110^\circ$,

$|ab|$ = 10 km and $|bc|$ = 20 km.

Calculate

(i) $|bp|$, correct to the nearest km

(ii) $|cp|$, correct to the nearest km.



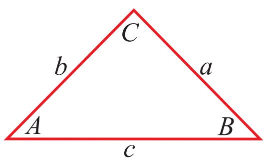
SOLUTION

5 (c) (i)

SINE RULE FORMULA

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$ .... **9** *OR* $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ **9**

You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



REMEMBER IT AS:

$$\frac{\sin(\text{Angle 1})}{\text{Opposite side}} = \frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad OR \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})} = \frac{\text{Opposite side}}{\sin(\text{Angle 2})}$$

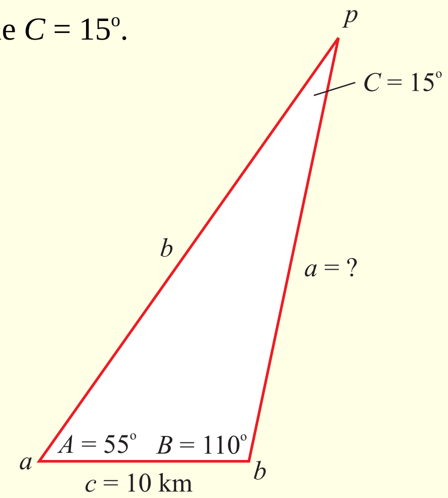
CONT....

The 3 angles of a triangle add up to 180°. Therefore, angle C = 15°.

$$\frac{a}{\sin A}=\frac{c}{\sin C}\Rightarrow\frac{a}{\sin 55^\circ}=\frac{10}{\sin 15^\circ}$$

$$\Rightarrow a=\frac{10\sin 55^\circ}{\sin 15^\circ}$$

$$\therefore a=|bp|=32\text{ km}$$



5 (c) (ii)

THE COSINE RULE

$$\boxed{a^2=b^2+c^2-2bc\cos A}\ \ldots\ldots\ \mathbf{10}$$

You use the Cosine rule when you are given:

[A] Two sides and one included angle,

[B] Three sides.

There are two other versions of the cosine rule not given in the tables:

$$\boxed{\begin{aligned}b^2&=a^2+c^2-2ac\cos B\\ c^2&=a^2+b^2-2ab\cos C\end{aligned}}\ \ldots\ldots\ \mathbf{10}$$

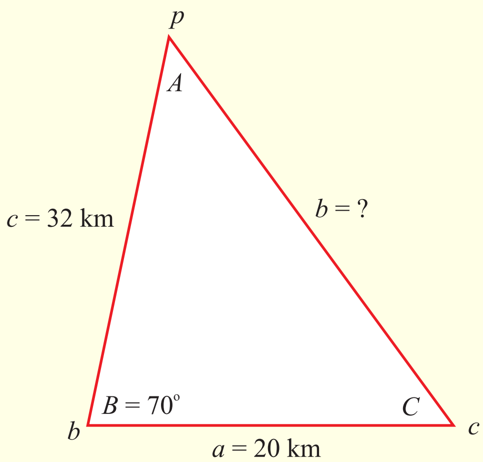
$|\angle pbc|=70^\circ$ [Straight angle]

$$b^2=a^2+c^2-2ac\cos B$$

$$\Rightarrow b^2=20^2+32^2-2(20)(32)\cos 70^\circ$$

$$\Rightarrow b^2=400+1024-1280\cos 70^\circ$$

$$\therefore b=|cp|=31\text{ km}$$



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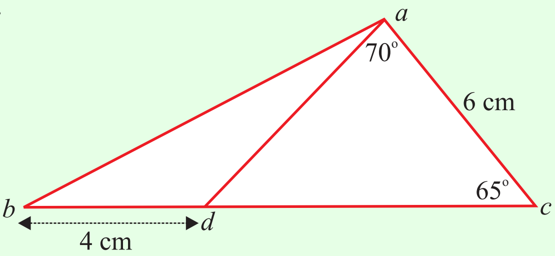
5 (c) *abc* is a triangle and $d \in [bc]$, as shown.

If $|bd| = 4$ cm, $|ac| = 6$ cm, $|\angle acd| = 65^\circ$

and $|\angle dac| = 70^\circ$, find

(i) $|dc|$, correct to the nearest cm

(ii) the area of triangle *abc*, correct to the nearest cm^2.



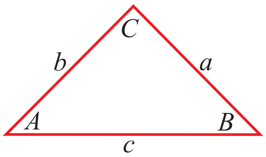
SOLUTION

5 (c) (i)

SINE RULE FORMULA

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}} \quad \text{.......}\ 9 \qquad OR \qquad \boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}} \quad \text{.......}\ 9$$

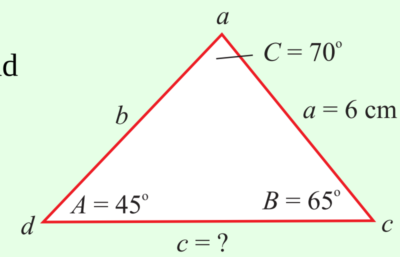
You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



REMEMBER IT AS:

$$\boxed{\frac{\sin(\text{Angle 1})}{\text{Opposite side}} = \frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad OR \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})} = \frac{\text{Opposite side}}{\sin(\text{Angle 2})}}$$

Consider triangle *adc*.
You can find angle A as the 3 angles in a triangle add up to 180°.



$$\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow \frac{c}{\sin 70^\circ} = \frac{6}{\sin 45^\circ}$$

$$\Rightarrow c = \frac{6 \sin 70^\circ}{\sin 45^\circ}$$

$$\therefore c = |dc| = 8 \text{ cm}$$

5 (c) (ii)

AREA OF A NON RIGHT-ANGLED TRIANGLE

$$\boxed{A = \tfrac{1}{2} ab \sin C} \quad \text{.......}\ 6$$

REMEMBER IT AS:

$$\boxed{\text{Area} = \tfrac{1}{2} \times \text{Product of 2 sides} \times \text{Sine of the included angle}}$$

$$A = \tfrac{1}{2}(6)(12)\sin 65^\circ$$

$$\therefore A = 33 \text{ cm}^2$$