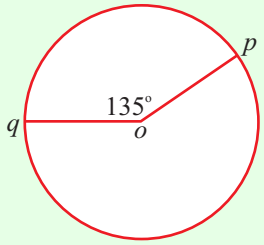
TIIGONOMETRY (Q 5, PAPER 2)

LESSON NO. 3: SECTOR OF A CIRCLE

2005

5 (a) A circle has centre o and radius 14 cm.
p and q are two points on the circle and
$|\angle qop| = 135^\circ$.
Find the length of the shorter arc pq.
Take $\pi = \frac{22}{7}$.



SOLUTION

$$s = 2\pi r \times \frac{\theta}{360^\circ}$$

$$\Rightarrow |pq| = 2 \times \frac{22}{7} \times 14 \times \frac{135^\circ}{360^\circ}$$

$$\therefore |pq| = 33 \text{ cm}$$

Length of arc

$$\boxed{s = 2\pi r \times \frac{\theta}{360^\circ}} \quad \ldots\ldots \; 7$$

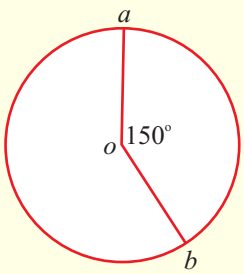
2004

5 (b) A circle has centre o and radius 4 cm.
a and b are two points on the circle and
$|\angle aob| = 150^\circ$.

(i) Find the area of the circle, correct to the nearest cm^2.

(ii) Find the area of the sector aob, correct to the nearest cm^2.

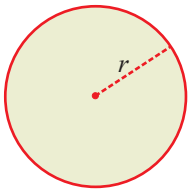
(iii) Find the length of the shorter arc ab, correct to the nearest cm.



SOLUTION

5 (b) (i)

6. CIRCLE



L: Length of Circumference
r: Radius

$$\boxed{L = 2\pi r} \quad \ldots\ldots \; 7$$

$$\boxed{A = \pi r^2} \quad \ldots\ldots \; 8$$

$r = 4$ cm

Area of circle: $A = \pi r^2 = \pi(4)^2 = 50 \text{ cm}^2$

5 (b) (ii)

Area of sector

$$\boxed{A = \pi r^2 \times \frac{\theta}{360^\circ}} \quad \ldots\ldots \; 8$$

Area of sector aob: $A = \pi r^2 \times \frac{\theta}{360^\circ} = 50 \times \frac{150}{360} = 21 \text{ cm}^2$

CONT....

5 (b) (iii)

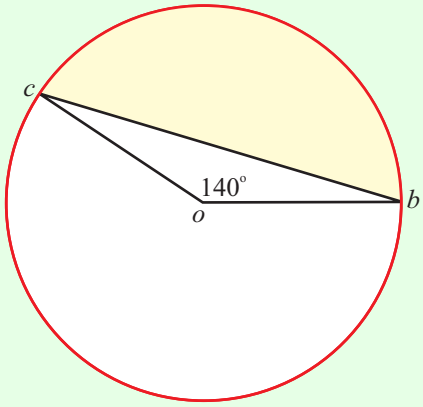
Length of arc

$$\boxed{s = 2\pi r \times \frac{\theta}{360^\circ}} \text{ ..... } 7$$

Length of shorter arc *aob*: $s = 2\pi r \times \dfrac{\theta}{360^\circ} = 2\pi(4) \times \dfrac{150}{360} = 10 \text{ cm}$

2003

5 (b) A circle has centre *o* and radius 7 cm. The two points *b* and *c* are on the circle and $|\angle boc| = 140^\circ$.



(i) Find the area of the triangle *obc*, correct to the nearest cm^2.

(ii) Find the area of the sector *obc*, correct to the nearest cm^2.

(iii) Taking the areas correct to the nearest cm^2, express the area of the shaded region as a fraction of the total area enclosed by the circle. Give your answer as a fraction in its simplest form.

SOLUTION

5 (b) (i)

AREA OF A NON RIGHT-ANGLED TRIANGLE

$$\boxed{A = \tfrac{1}{2}ab\sin C} \text{ ..... } 6$$

REMEMBER IT AS:

$$\boxed{\text{Area} = \tfrac{1}{2} \times \text{Product of 2 sides} \times \text{Sine of the included angle}}$$

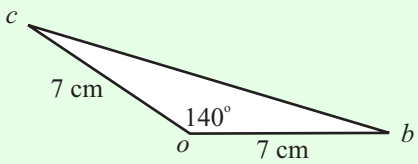
Area of triangle *obc*:

Area $= \frac{1}{2} \times$ Product of 2 sides $\times$ Sine of the included angle

$\Rightarrow A = \frac{1}{2}(7)(7)\sin 140^\circ$

$\Rightarrow A = \frac{49}{2}\sin 140^\circ$

$\therefore A = 16 \text{ cm}^2$



5 (b) (ii)

Area of sector *obc*: $A = \pi r^2 \times \dfrac{\theta}{360^\circ} \Rightarrow A = \pi(7)^2 \times \dfrac{140^\circ}{360^\circ}$

$\therefore A = 60 \text{ cm}^2$

Area of sector

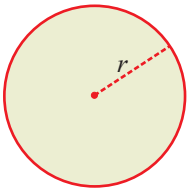
$$\boxed{A = \pi r^2 \times \frac{\theta}{360^\circ}} \text{ ..... } 8$$

CONT....

5 (b) (iii)

Shaded area = Area of sector *obc* – Area of triangle *obc* = 60 – 16 = 44 cm²

6. **Circle**



L: Length of Circumference
r: Radius

$L = 2\pi r$ 7

$A = \pi r^2$ 8

Area of circle: $A = \pi r^2 \Rightarrow A = \pi(7)^2 = 49\pi$

$\therefore A = 154 \text{ cm}^2$

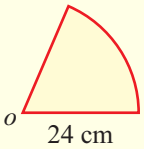
$$\frac{\text{Area of shaded region}}{\text{Area of circle}} = \frac{44}{154} = \frac{2}{7}$$

2002

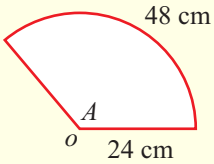
5 (b) A circle has radius 24 cm and centre *o*.

(i) Calculate the area of a sector which has 70° at *o*.

Take $\pi = \frac{22}{7}$.



(ii) An arc of length 48 cm subtends an angle *A* at *o*.
Calculate *A*, correct to the nearest degree.

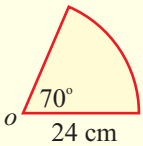


Solution

5 (b) (i)

$$A = \pi r^2 \times \frac{\theta}{360^\circ} \Rightarrow A = \left(\frac{22}{7}\right)(24)^2\left(\frac{70^\circ}{360^\circ}\right)$$

$\therefore A = 352 \text{ cm}^2$



Area of sector

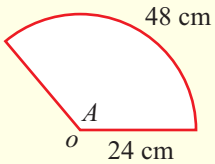
$A = \pi r^2 \times \frac{\theta}{360^\circ}$ 8

5 (b) (ii)

$$s = 2\pi r \times \frac{\theta}{360^\circ} \Rightarrow 48 = 2\pi(24)\left(\frac{A}{360^\circ}\right)$$

$$\Rightarrow 48 = 48\pi\left(\frac{A}{360^\circ}\right)$$

$$\therefore A = \frac{360}{\pi} = 115^\circ$$



Length of arc

$s = 2\pi r \times \frac{\theta}{360^\circ}$ 7

1999

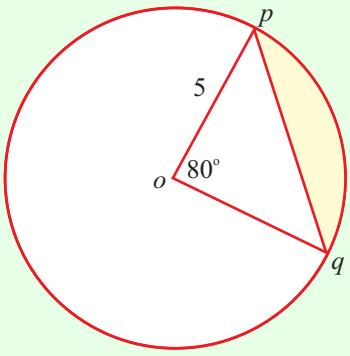
5 (b) In the diagram, o is the centre of the circle with radius length 5 and p and q are points on the circle.

$|\angle poq| = 80^\circ$.

Find, correct to two places of decimals,

(i) the area of triangle poq

(ii) the area of the shaded region, taking $\pi = 3{\cdot}14$.



SOLUTION

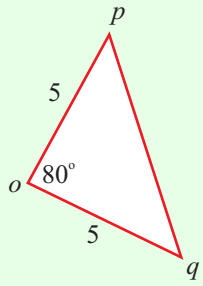
5 (b) (i)

AREA OF A NON RIGHT-ANGLED TRIANGLE

$A = \frac{1}{2}ab\sin C$ **6**

REMEMBER IT AS:

Area = $\frac{1}{2}\times$ Product of 2 sides $\times$ Sine of the included angle



$A = \frac{1}{2}(5)(5)\sin 80^\circ = 12.31$ square units

5 (b) (ii)

Area of shaded region = Area of sector poq – Area if triangle poq

Area of sector poq: $A = \pi r^2 \times \dfrac{\theta}{360^\circ} \Rightarrow A = (3.14)(5)^2 \times \dfrac{80^\circ}{360^\circ}$

$\therefore A = 17.44$ square units

Area of sector

$A = \pi r^2 \times \dfrac{\theta}{360^\circ}$ **8**

Area of shaded region = 17.44 – 12.31 = 5.13 square units

1998

5 (a) The angle at the centre of a sector of a disc measures 140°.

The radius of the disc measures 6 cm.

Find, in terms of π, the area of the sector.

SOLUTION

5 (a)

$A = \pi r^2 \times \dfrac{\theta}{360^\circ} = \pi(6)^2 \times \dfrac{140^\circ}{360^\circ}$

$\therefore A = 14\pi \text{ cm}^2$

Area of sector

$A = \pi r^2 \times \dfrac{\theta}{360^\circ}$ **8**

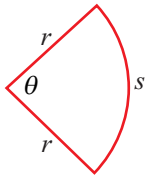
1996

5 (a) Find the length of an arc of a circle of radius length 6 cm subtending an angle of 120° at the centre. Give your answer in terms of π.

SOLUTION

5 (a)

Length of arc: $\boxed{s = 2\pi r \times \dfrac{\theta}{360^{\circ}}}$ 7



$$s = 2\pi r \times \frac{\theta}{360^{\circ}} = 2\pi(6) \times \frac{120^{\circ}}{360^{\circ}}$$

$\therefore s = 4\pi$ cm