Tiigonometry (Q 5, Paper 2)

Lesson No. 1: Right-angled Triangles

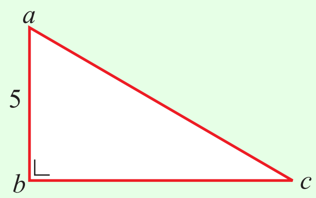
2007

5 (b) In the right-angled triangle abc, $|ab| = 5$ cm.
The area of the triangle is 15 cm^2.

(i) Find $|bc|$.

(ii) Find $|\angle cab|$, correct to the nearest degree.

(iii) Find $|\angle bca|$, correct to the nearest degree.



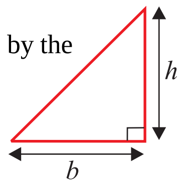
Solution

5 (b) (i)

Area of a right-angled triangle

You can find the area, A, by multiplying half the base, b, by the perpendicular height, h.

$\boxed{A = \frac{1}{2}bh}$ 2



$$A = \tfrac{1}{2}|bc||ab| \Rightarrow 15 = \tfrac{1}{2}|bc|(5)$$

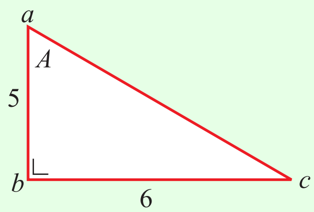
$$\therefore |bc| = \frac{2 \times 15}{5} = 6 \text{ cm}$$

5 (b) (ii)

$\boxed{\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}}$ 5

$$\tan A = \tfrac{6}{5} \Rightarrow A = \tan^{-1}(\tfrac{6}{5})$$

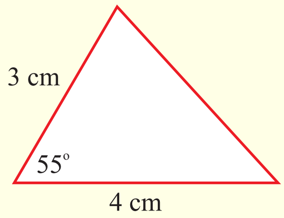
$$\therefore A = |\angle cab| = 50^\circ$$



2006

5 (a) The lengths of two sides of a right-angled triangle are shown in the diagram.

(i) Copy the diagram into your answer book and on it mark the angle A such that $\tan A = \frac{5}{8}$.

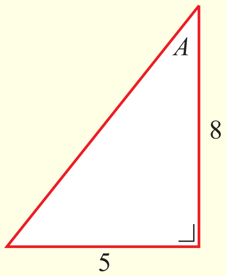


(ii) Find the area of the triangle.

SOLUTION

5 (a) (i)

$\boxed{\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}}$ **5**

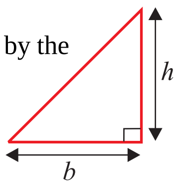


5 (a) (ii)

AREA OF A RIGHT-ANGLED TRIANGLE

You can find the area, A, by multiplying half the base, b, by the perpendicular height, h.

$\boxed{A = \frac{1}{2}bh}$ **2**



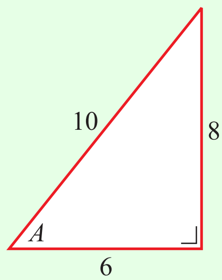
$A = \frac{1}{2}(5)(8) = 20$ square units

2004

5 (a) The lengths of the sides of a right-angled triangle are shown in the diagram and A is the angle indicated.

(i) Write down the value of $\cos A$.

(ii) Hence, find the angle A, correct to the nearest degree.



SOLUTION

5 (a) (i)

$\boxed{\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}}$ **3**

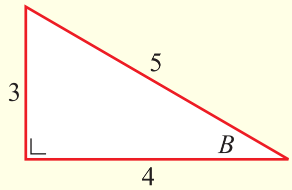
$\cos A = \frac{6}{10} = \frac{3}{5}$

5 (a) (ii)

$\cos A = \frac{3}{5} \Rightarrow A = \cos^{-1}(\frac{3}{5}) = 53^\circ$

2003

5 (a) The lengths of the sides of a right-angled triangle are shown in the diagram and B is the angle indicated. Find the value of sin Bcos B, as a fraction.

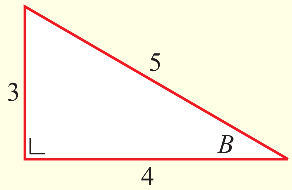


Solution

$\sin B = \frac{3}{5}$

$\cos B = \frac{4}{5}$

$\therefore \sin B \cos B = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$



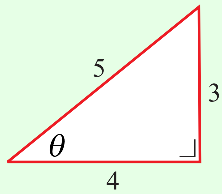
$\boxed{\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}}$ 3

$\boxed{\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}}$ 4

2002

5 (a) Use the information given in the diagram to show that

$$\sin\theta + \cos\theta > \tan\theta.$$



Solution

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

$\sin\theta + \cos\theta = \frac{3}{5} + \frac{4}{5} = \frac{7}{5} > \frac{3}{4}$

$\therefore \sin\theta + \cos\theta > \tan\theta$

$\boxed{\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}}$ 3

$\boxed{\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}}$ 4

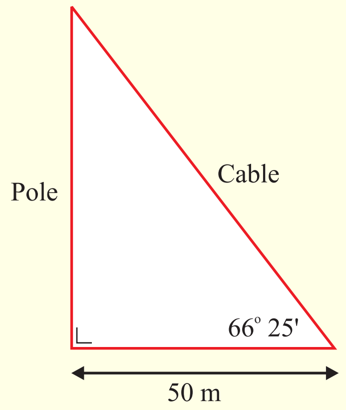
$\boxed{\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}}$ 5

2000

5 (b) The diagram shows a vertical pole which stands on level ground.
A cable joins the top of the pole to a point on the ground which is 50 m from the base of the pole.
The cable makes an angle of $66^\circ 25'$ with the ground.

(i) Find the height of the pole, correct to the nearest metre.

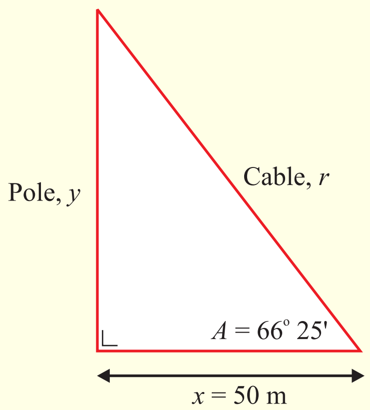
(ii) Find the length of the cable, correct to the nearest metre.



SOLUTION

5 (b) (i)

$$\boxed{\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}} \;\ldots\ldots\; 5$$



$$\tan A = \frac{y}{x} \Rightarrow y = x \tan A$$

$$\Rightarrow y = 50 \tan 66^\circ 25'$$

$\therefore$ Height of pole $y = 115$ m

5 (b) (i)

$$\boxed{\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}} \;\ldots\ldots\; 4$$

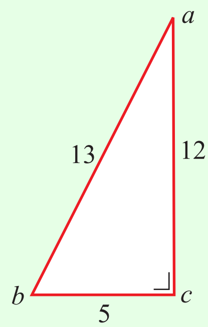
$$\sin A = \frac{y}{r} \Rightarrow r = \frac{y}{\sin A}$$

$$\Rightarrow r = \frac{115}{\sin 66^\circ 25'}$$

$\therefore$ Length of cable $r = 125$ m

1999

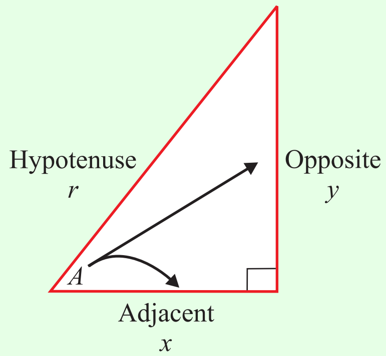
5 (a) abc is a right-angled triangle with $|\angle acb| = 90^\circ$,
$|ab| = 13$, $|bc| = 5$ and $|ac| = 12$.
Find, as fractions, the value of $\sin\angle abc$ and the value of $\tan\angle bac$.



SOLUTION

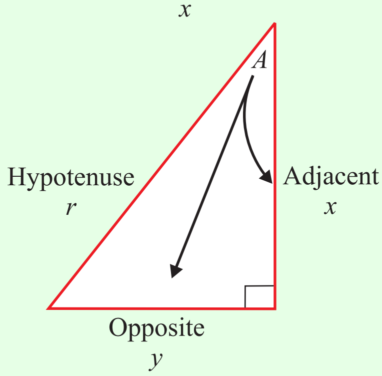
$\boxed{\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}}$ 4

$\sin\angle abc = \frac{12}{13}$



$\boxed{\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}}$ 5

$\tan\angle bac = \frac{5}{12}$



1998

5 (b) A is an acute angle such that $\tan A = \frac{21}{20}$.

(i) Find, as fractions, the value of $\cos A$ and the value of $\sin A$.

(ii) Find the measurement of angle A, correct to the nearest degree.

Solution

5 (b) (i)

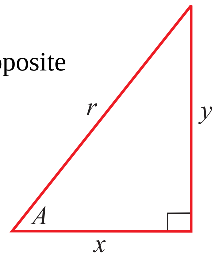
$\boxed{\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}}$ 5

Pythagoras

One of the angles in a right-angled triangle is 90°. The side opposite this angle is called the **hypotenuse**.

Pythagoras' theorem applies to right-angled triangles.

$\boxed{x^2 + y^2 = r^2}$ 1



$x^2 + y^2 = r^2 \Rightarrow 20^2 + 21^2 = r^2$

$\Rightarrow r^2 = 400 + 441 = 841$

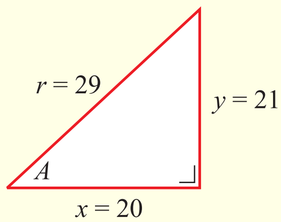
$\therefore r = \sqrt{841} = 29$

$\cos A = \frac{20}{29}$

$\sin A = \frac{21}{29}$

$\boxed{\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}}$ 3

$\boxed{\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}}$ 4



5 (b) (ii)

$\tan A = \frac{21}{20} \Rightarrow A = \tan^{-1}(\frac{21}{20})$

$\therefore A = 46^\circ$