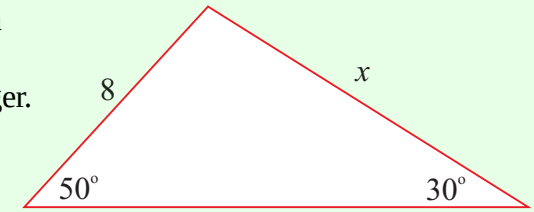
TRIGONOMETRY (Q 5, PAPER 2)

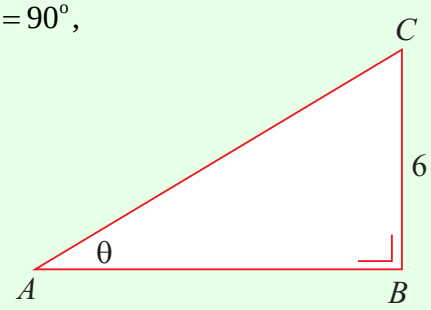
2011

5. **(a)** Use the sine rule to calculate the value of x in the diagram.
Give your answer correct to the nearest integer.



(b) In the triangle ABC, $|BC| = 6$ cm, and $|\angle ABC| = 90°$,

$|\angle CAB| = \theta$ and $\sin\theta = \frac{3}{5}$.



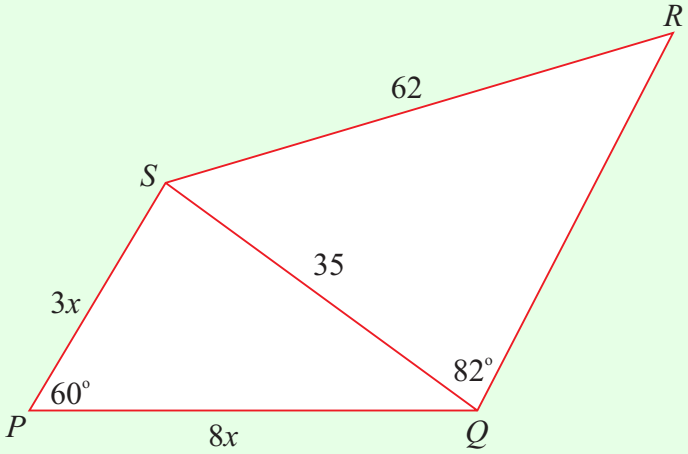
(i) Find $|AC|$.

(ii) Find $|AB|$.

(iii) Verify that $\cos^2\theta + \sin^2\theta = 1$.

(c) $PQRS$ is a quadrilateral with diagonal $[SQ]$.

$|RS| = 62$, $|SQ| = 35$, $|\angle SQR| = 82°$, $|\angle SPQ| = 60°$, $|SP| = 3x$ and $|PQ| = 8x$.



(i) Find $|\angle QRS|$, correct to the nearest degree, given that $0° \le |\angle QRS| \le 90°$.

(ii) Find the value of x.

SOLUTION

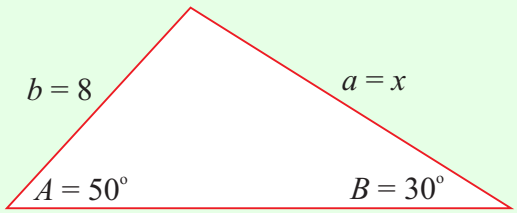
5 (a)

$$\boxed{\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}}$$

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$

$$\frac{x}{\sin 50^\circ}=\frac{8}{\sin 30^\circ}$$

$$\therefore x=\frac{8\sin 50^\circ}{\sin 30^\circ}=12$$



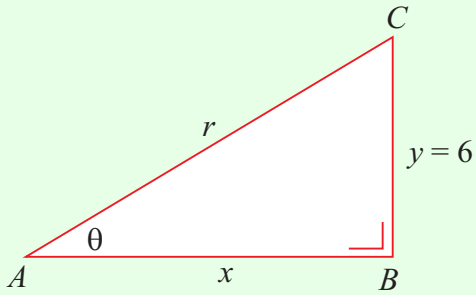
5 (b) (i)

$$\sin\theta=\frac{3}{5}=\frac{6}{|AC|}$$

$$\boxed{\sin A=\frac{y}{r}=\frac{\text{Opposite}}{\text{Hypotenuse}}}$$

$$3|AC|=30$$

$$|AC|=10\text{ cm}$$



5 (b) (ii)

$$|AB|^2+|BC|^2=|AC|^2 \qquad \boxed{x^2+y^2=r^2}$$

$$|AB|^2+6^2=10^2$$

$$|AB|^2+36=100$$

$$|AB|^2=100-36$$

$$|AB|^2=64$$

$$|AB|=\sqrt{64}=8\text{ cm}$$

5 (b) (iii)

$$\sin\theta=\tfrac{3}{5}$$

$$\cos\theta=\tfrac{8}{10}=\tfrac{4}{5} \qquad \boxed{\cos A=\frac{x}{r}=\frac{\text{Adjacent}}{\text{Hypotenuse}}}$$

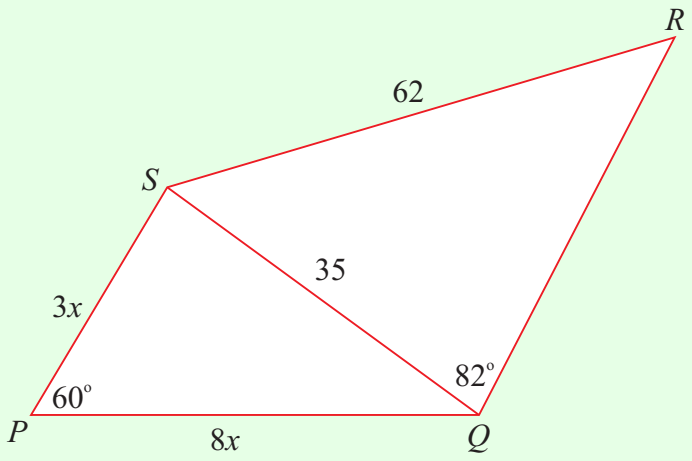
$$\cos^2\theta+\sin^2\theta=(\tfrac{4}{5})^2+(\tfrac{3}{5})^2$$

$$=\tfrac{16}{25}+\tfrac{9}{25}$$

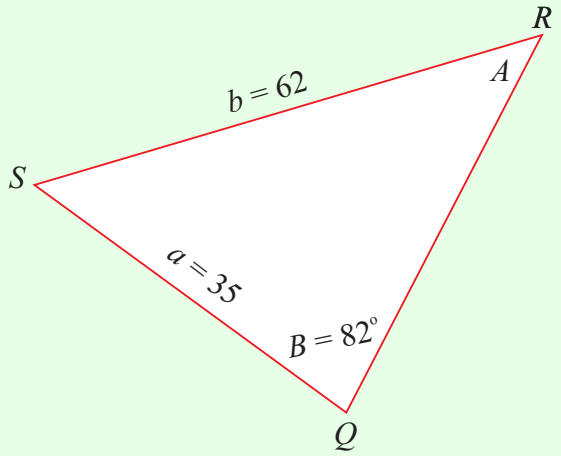
$$=\tfrac{25}{25}$$

$$=1$$

5 (c) (i)



Separate out the triangles. Consider triangle *SRQ*. Call *A* angle $|\angle QRS|$.



$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{35} = \frac{\sin 82^\circ}{62}$$

$$\sin A = \frac{35 \sin 82^\circ}{62}$$

$$\therefore A = \sin^{-1}\left(\frac{35 \sin 82^\circ}{62}\right) = 34^\circ$$

5 (c) (ii)

Consider triangle *SPQ*.

$$\boxed{a^2 = b^2 + c^2 - 2bc \cos A}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$35^2 = (8x)^2 + (3x)^2 - 2(8x)(3x)\cos 60^\circ$$

$$1225 = 64x^2 + 9x^2 - 48x^2\left(\tfrac{1}{2}\right)$$

$$1225 = 64x^2 + 9x^2 - 24x^2$$

$$1225 = 49x^2$$

$$\frac{1225}{49} = x^2$$

$$25 = x^2$$

$$\sqrt{25} = x$$

$$\therefore x = 5$$