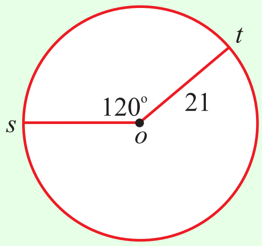
TRIGONOMETRY (Q 5, PAPER 2)

2008

5 (a) A circle has centre o and radius 21 cm.

s and t are two points on the circle and $|\angle sot| = 120^\circ$.

Find the length of the shorter arc st, correct to the nearest centimetre.

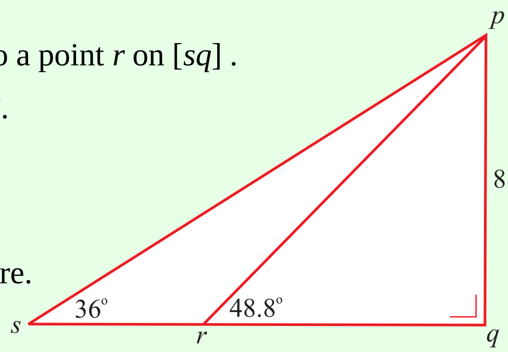


(b) In the right-angled triangle psq, p is joined to a point r on $[sq]$.

$|pq| = 8$ cm, $|\angle prq| = 48.8^\circ$ and $|\angle psq| = 36^\circ$.

(i) Find $|pr|$, correct to one decimal place.

(ii) Find $|sr|$, correct to the nearest centimetre.

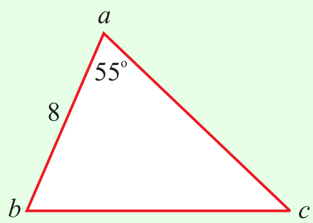


(c) The area of the triangle abc is 33 cm².

$|ab| = 8$ cm and $|\angle cab| = 55^\circ$.

(i) Find $|bc|$, correct to one decimal place.

(ii) Find $|\angle abc|$, correct to the nearest degree.



SOLUTION

5 (a)

$r = 21$ cm, $\theta = 120^\circ$

$$|st| = 2\pi(21) \times \frac{120^\circ}{360^\circ} = 14\pi = 44 \text{ cm}$$

Length of arc

$$\boxed{s = 2\pi r \times \frac{\theta}{360^\circ}}$$ 7

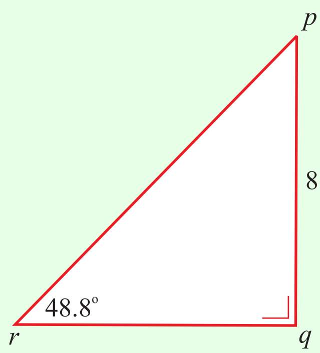
5 (b) (i)

Consider the right-angled triangle pqr.

$$\sin 48.8^\circ = \frac{8}{|pr|}$$

$$\boxed{\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}}$$ 4

$$\Rightarrow |pr| = \frac{8}{\sin 48.8^\circ} = 10.6 \text{ cm}$$

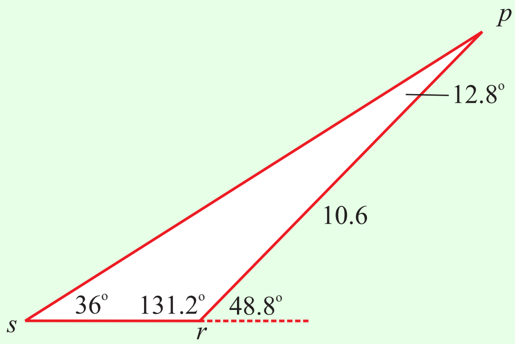


5 (b) (ii)

Consider triangle *psr*. Work out all of its angles.

$\angle prs = 180^\circ - 48.8^\circ = 131.2^\circ$

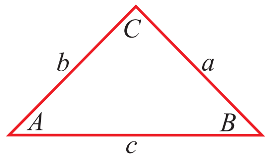
$\angle rps = 180^\circ - 131.2^\circ - 36^\circ = 12.8^\circ$



SINE RULE FORMULA

$\boxed{\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}}$ .... **9** *OR* $\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$ **9**

You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



REMEMBER IT AS:

$\boxed{\dfrac{\sin(\text{Angle 1})}{\text{Opposite side}} = \dfrac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad OR \quad \dfrac{\text{Opposite side}}{\sin(\text{Angle 1})} = \dfrac{\text{Opposite side}}{\sin(\text{Angle 2})}}$

$$\frac{|sr|}{\sin 12.8^\circ} = \frac{10.6}{\sin 36^\circ}$$

$$\Rightarrow |sr| = \frac{10.6 \sin 12.8^\circ}{\sin 36^\circ} = 4 \text{ cm}$$

5 (c) (i)

AREA OF A NON RIGHT-ANGLED TRIANGLE

$\boxed{A = \tfrac{1}{2} ab \sin C}$ **6**

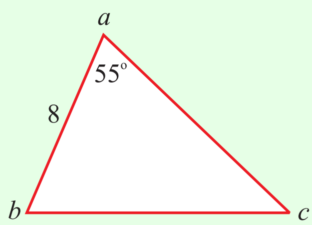
REMEMBER IT AS:

$\boxed{\text{Area} = \tfrac{1}{2} \times \text{Product of 2 sides} \times \text{Sine of the included angle}}$

$$33 = \tfrac{1}{2} \times 8 \times |ac| \times \sin 55^\circ$$

$$\Rightarrow 33 = 4 \times |ac| \times \sin 55^\circ$$

$$\Rightarrow |ac| = \frac{33}{4 \sin 55^\circ} = 10.07 \text{ cm}$$



Now use the Cosine rule to find $|bc|$.

THE COSINE RULE

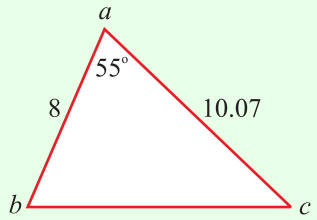
$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$ **10**

You use the Cosine rule when you are given:
[A] Two sides and one included angle,
[B] Three sides.

There are two other versions of the cosine rule not given in the tables:

$$\boxed{\begin{array}{l} b^2 = a^2 + c^2 - 2ac\cos B \\ c^2 = a^2 + b^2 - 2ab\cos C \end{array}}$$ **10**

$$|bc|^2 = 8^2 + 10.07^2 - 2(8)(10.07)\cos 55^\circ$$
$$\Rightarrow |bc|^2 = 73$$
$$\therefore |bc| = \sqrt{73} = 8.5 \text{ cm}$$



5 (c) (ii)

Let $|\angle abc| = B.$

$$\frac{\sin B}{10.07} = \frac{\sin 55^\circ}{8.5}$$

$$\Rightarrow B = \sin^{-1}\left(\frac{10.07\sin 55^\circ}{8.5}\right) = 76^\circ$$