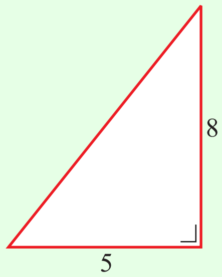
TRIGONOMETRY (Q 5, PAPER 2)

2006

5 (a) The lengths of two sides of a right-angled triangle are shown in the diagram.

(i) Copy the diagram into your answer book and on it mark the angle A such that $\tan A = \frac{5}{8}$.

(ii) Find the area of the triangle.

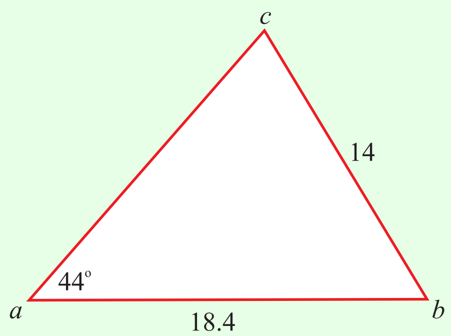


(b) In the triangle abc,

$|ab| = 18.4$, $|bc| = 14$ and $|\angle cab| = 44^\circ$.

(i) Find $|\angle bca|$, correct to the nearest degree.

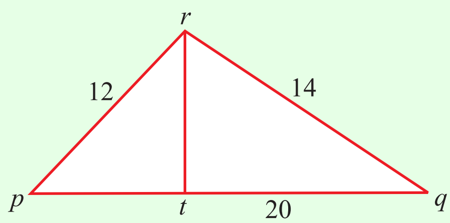
(ii) Find the area of the triangle abc, correct to the nearest whole number.



(c) The lengths of the sides of the triangle pqr are $|pq| = 20$, $|qr| = 14$ and $|pr| = 12$.

(i) Find $|\angle rpq|$, correct to one decimal place.

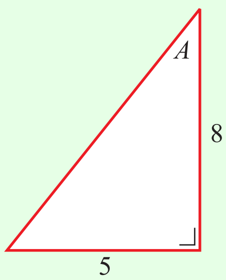
(ii) Find $|rt|$, where $rt \perp pq$. Give your answer correct to the nearest whole number.



SOLUTION

5 (a) (i)

$$\boxed{\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}} \;\ldots\ldots\; 5$$

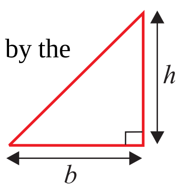


5 (a) (ii)

AREA OF A RIGHT-ANGLED TRIANGLE

You can find the area, A, by multiplying half the base, b, by the perpendicular height, h.

$$\boxed{A = \tfrac{1}{2}bh} \;\ldots\ldots\; 2$$



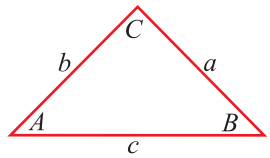
$A = \frac{1}{2}(5)(8) = 20$ square units

5 (b) (i)

SINE RULE FORMULA

$$\boxed{\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}} \;\ldots\ldots\; 9 \quad \textit{OR} \quad \boxed{\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}} \;\ldots\ldots\; 9$$

You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



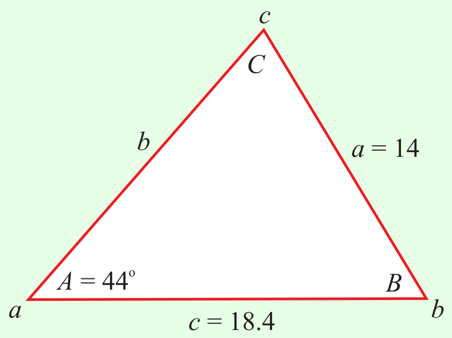
REMEMBER IT AS:

$$\boxed{\frac{\sin(\text{Angle 1})}{\text{Opposite side}}=\frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad \textit{OR} \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})}=\frac{\text{Opposite side}}{\sin(\text{Angle 2})}}$$

$$\frac{\sin C}{c}=\frac{\sin A}{a} \Rightarrow \frac{\sin C}{18.4}=\frac{\sin 44^\circ}{14}$$

$$\Rightarrow \sin C=\frac{18.4\sin 44^\circ}{14}=0.913$$

$$\therefore C=|\angle bca|=\sin^{-1}(0.913)=66^\circ$$



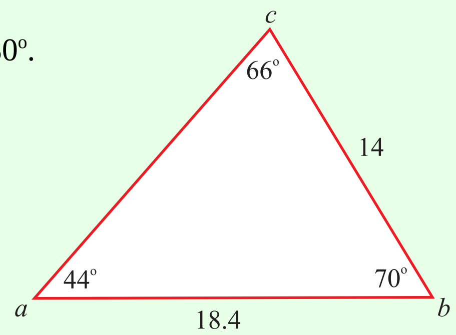
5 (b) (ii)

The three angles in a triangle add up to 180°.
Find the third angle in the triangle.

$$|\angle abc|+44^\circ+66^\circ=180^\circ$$

$$\Rightarrow |\angle abc|=180^\circ-44^\circ-66^\circ$$

$$\therefore |\angle abc|=70^\circ$$



AREA OF A NON RIGHT-ANGLED TRIANGLE

$$\boxed{A=\tfrac{1}{2}ab\sin C} \;\ldots\ldots\; 6$$

REMEMBER IT AS:

$$\boxed{\text{Area} = \tfrac{1}{2}\times \text{Product of 2 sides} \times \text{Sine of the included angle}}$$

$$A=\tfrac{1}{2}(14)(18.4)\sin 70^\circ=121 \text{ square units}$$

5 (c) (i)

THE COSINE RULE

$a^2 = b^2 + c^2 - 2bc\cos A$ **10**

You use the Cosine rule when you are given:
[A] Two sides and one included angle,
[B] Three sides.

There are two other versions of the cosine rule not given in the tables:

$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$ **10**

$a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow 14^2 = 12^2 + 20^2 - 2(12)(20)\cos A$

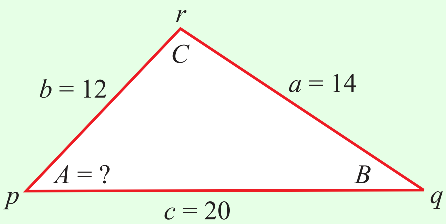
$\Rightarrow 196 = 144 + 400 - 480\cos A$

$\Rightarrow 480\cos A = 144 + 400 - 196$

$\Rightarrow 480\cos A = 348$

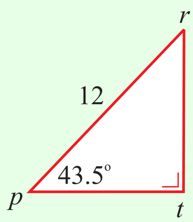
$\Rightarrow \cos A = \dfrac{348}{480}$

$\therefore A = |\angle rpq| = \cos^{-1}\left(\dfrac{348}{480}\right) = 43.5^\circ$



5 (c) (ii)

$\sin A = \dfrac{y}{r} = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$ **4**



$\sin 43.5^\circ = \dfrac{|rt|}{12} \Rightarrow |rt| = 12\sin 43.5^\circ$

$\therefore |rt| = 8$