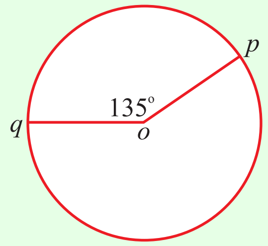
TRIGONOMETRY (Q 5, PAPER 2)

2005

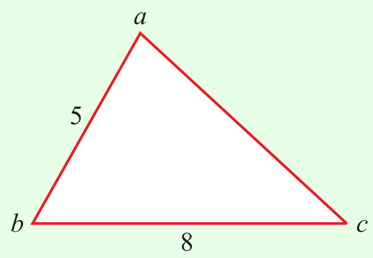
5 (a) A circle has centre o and radius 14 cm.
p and q are two points on the circle and
$|\angle qop| = 135^\circ$.
Find the length of the shorter arc pq.
Take $\pi = \frac{22}{7}$.



(b) In the triangle abc, $|ab| = 5$ cm and $|bc| = 8$ cm.
The area of the triangle is 16.58 cm^2.

(i) Find $|\angle abc|$, correct to the nearest degree.

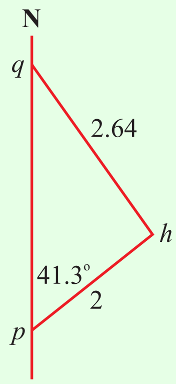
(ii) Find $|ac|$, correct to the nearest centimetre.



(c) A lighthouse, h, is observed from a ship sailing a straight course due North.
The distance from p to h is 2 km and the bearing of the lighthouse from p is N 41.3° E.
The distance from q to h is 2.64 km.

(i) Find the bearing of the lighthouse from q.

(ii) The ship is sailing at a speed of 19 km/h. Find, correct to the nearest minute, the time taken to sail from p to q.



SOLUTION

5 (a)

Length of arc

$$\boxed{s = 2\pi r \times \frac{\theta}{360^\circ}} \;\cdots\cdots\; 7$$

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$$\Rightarrow |pq| = 2 \times \frac{22}{7} \times 14 \times \frac{135^\circ}{360^\circ}$$

$$\therefore |pq| = 33 \text{ cm}$$

5 (b) (i)

AREA OF A NON RIGHT-ANGLED TRIANGLE

$\boxed{A = \frac{1}{2}ab\sin C}$ **6**

REMEMBER IT AS:

$\boxed{\text{Area} = \frac{1}{2} \times \text{Product of 2 sides} \times \text{Sine of the included angle}}$

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$$\Rightarrow 16.58 = \tfrac{1}{2}(5)(8)\sin|\angle abc|$$

$$\Rightarrow 16.58 = 20\sin|\angle abc|$$

$$\Rightarrow \sin|\angle abc| = \frac{16.58}{20}$$

$$\therefore |\angle abc| = \sin^{-1}\left(\frac{16.58}{20}\right) = 56^\circ$$

5 (b) (ii)

THE COSINE RULE

$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ **10**

You use the Cosine rule when you are given:
[A] Two sides and one included angle,
[B] Three sides.

There are two other versions of the cosine rule not given in the tables:

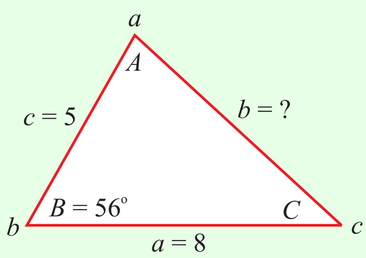
$$\boxed{\begin{aligned} b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$ **10**

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\Rightarrow b^2 = 8^2 + 5^2 - 2(8)(5)\cos 56^\circ$$

$$\Rightarrow b^2 = 64 + 25 - 80\cos 56^\circ$$

$$\therefore b = |ac| = 7 \text{ cm}$$

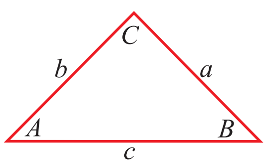


5 (c) (i)

SINE RULE FORMULA

$\boxed{\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}}$ ....**9** *OR* $\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$ **9**

You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



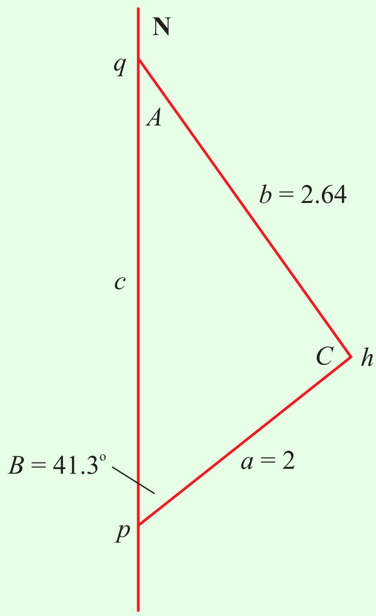
REMEMBER IT AS:

$$\boxed{\frac{\sin(\text{Angle 1})}{\text{Opposite side}} = \frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad OR \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})} = \frac{\text{Opposite side}}{\sin(\text{Angle 2})}}$$

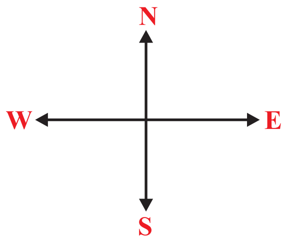
$$\frac{\sin A}{a}=\frac{\sin B}{b}\Rightarrow\frac{\sin A}{2}=\frac{\sin 41.3^{\circ}}{2.64}$$

$$\Rightarrow \sin A=\frac{2\sin 41.3^{\circ}}{2.64}=0.5$$

$$\therefore A=\sin^{-1}(0.5)=30^{\circ}$$

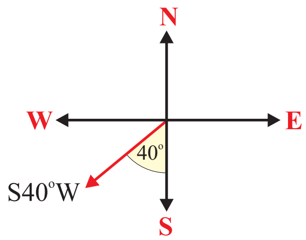


Compass Directions



The compass directions are shown. The four main directions are North (**N**), South (**S**), East (**E**) and West (**W**). Other directions are combinations of these.

Ex. S40°W means start at a point on the Southern direction and go 40° towards the West.



The bearing of the lighthouse from q is S 30° E.

5 (c) (ii)

Find the angle C. The 3 angles of a triangle add up to 180°.

$$C+41.3^{\circ}+30^{\circ}=180^{\circ}\Rightarrow C=108.7^{\circ}$$

Use the Sine Rule to find the distance $|qp|$.

$$\frac{c}{\sin C}=\frac{a}{\sin A}\Rightarrow\frac{c}{\sin 108.7^{\circ}}=\frac{2}{\sin 30^{\circ}}$$

$$\Rightarrow c=\frac{2\sin 108.7^{\circ}}{\sin 30^{\circ}}$$

$$\therefore c=|qp|=3.8 \text{ km}$$

$$\text{Speed }(v)=\frac{\text{Distance }(s)}{\text{Time }(t)} \qquad \boxed{v=\frac{s}{t}} \;\ldots\ldots\; 4$$

$$v=19 \text{ km/h}$$

$$s=3.8 \text{ km}$$

$$t=?$$

$$v=\frac{s}{t}\Rightarrow 19=\frac{3.8}{t}$$

$$\Rightarrow t=\frac{3.8}{19}=0.2 \text{ h}=12 \text{ minutes}$$