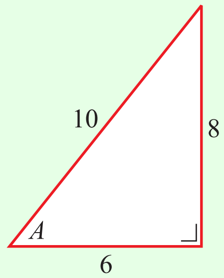
TRIGONOMETRY (Q 5, PAPER 2)

2004

5 (a) The lengths of the sides of a right-angled triangle are shown in the diagram and A is the angle indicated.

(i) Write down the value of cos A.

(ii) Hence, find the angle A, correct to the nearest degree.



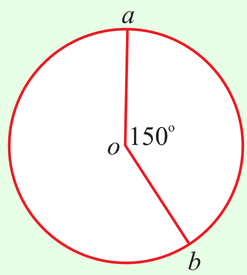
(b) A circle has centre o and radius 4 cm.
a and b are two points on the circle and

$|\angle aob| = 150^\circ$.

(i) Find the area of the circle, correct to the nearest cm^2.

(ii) Find the area of the sector aob, correct to the nearest cm^2.

(iii) Find the length of the shorter arc ab, correct to the nearest cm.

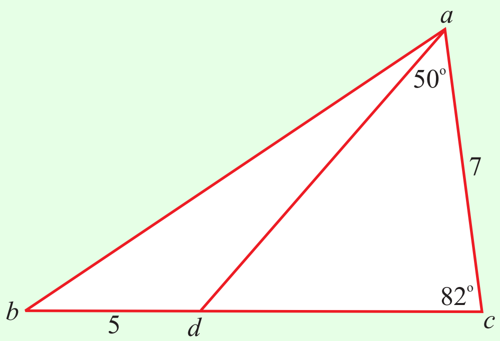


(c) In the triangle abc, d is a point on $[bc]$.
$|bd| = 5$ cm, $|ac| = 7$ cm,

$|\angle dca| = 82^\circ$ and $|\angle cad| = 50^\circ$.

(i) Find $|dc|$, correct to the nearest cm.

(ii) Find $|ab|$, correct to the nearest cm.



SOLUTION

5 (a) (i)

$$\boxed{\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}} \;\ldots\ldots\; 3$$

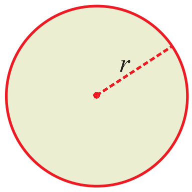
$\cos A = \frac{6}{10} = \frac{3}{5}$

5 (a) (ii)

$\cos A = \frac{3}{5} \Rightarrow A = \cos^{-1}(\frac{3}{5}) = 53^\circ$

5 (b) (i)

6. **Circle**



L: Length of Circumference
r: Radius

$\boxed{L = 2\pi r}$ 7

$\boxed{A = \pi r^2}$ 8

$r = 4$ cm

Area of circle: $A = \pi r^2 = \pi(4)^2 = 50 \text{ cm}^2$

5 (b) (ii)

Area of sector

$\boxed{A = \pi r^2 \times \dfrac{\theta}{360^\circ}}$ 8

Area of sector *aob*: $A = \pi r^2 \times \dfrac{\theta}{360^\circ} = 50 \times \dfrac{150}{360} = 21 \text{ cm}^2$

5 (b) (iii)

Length of arc

$\boxed{s = 2\pi r \times \dfrac{\theta}{360^\circ}}$ 7

Length of shorter arc *aob*: $s = 2\pi r \times \dfrac{\theta}{360^\circ} = 2\pi(4) \times \dfrac{150}{360} = 10 \text{ cm}$

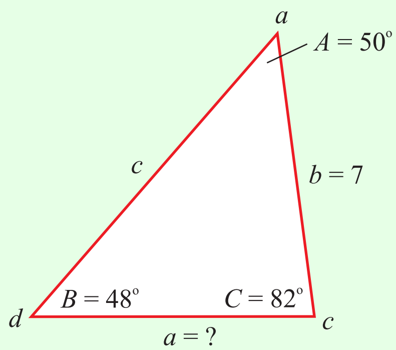
5 (c) (i)

Consider the triangle *adc*.
The 3 angles add up to 180° so you can calculate angle *B*.

$B + 50^\circ + 82^\circ = 180^\circ$

$\Rightarrow B = 180^\circ - 50^\circ - 82^\circ$

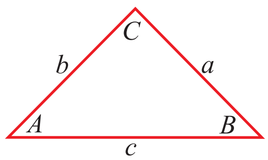
$\therefore B = 48^\circ$



Sine Rule Formula

$\boxed{\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}}$ .... 9 *OR* $\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$ 9

You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



Remember it as:

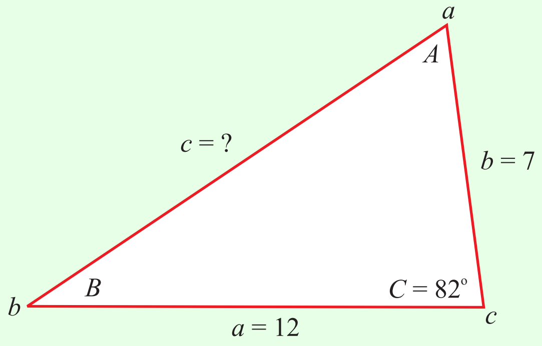
$\boxed{\dfrac{\sin(\text{Angle 1})}{\text{Opposite side}} = \dfrac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad OR \quad \dfrac{\text{Opposite side}}{\sin(\text{Angle 1})} = \dfrac{\text{Opposite side}}{\sin(\text{Angle 2})}}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{a}{\sin 50^\circ} = \frac{7}{\sin 48^\circ}$$

$$\Rightarrow a = |dc| = \frac{7 \sin 50^\circ}{\sin 48^\circ} = 7 \text{ cm}$$

5 (c) (ii)

Consider the triangle *abc*.



The Cosine Rule

$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ **10**

You use the Cosine rule when you are given:
[A] Two sides and one included angle,
[B] Three sides.

There are two other versions of the cosine rule not given in the tables:

$$\boxed{\begin{array}{l} b^2 = a^2 + c^2 - 2ac\cos B \\ c^2 = a^2 + b^2 - 2ab\cos C \end{array}}$$ **10**

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\Rightarrow c^2 = 12^2 + 7^2 - 2(12)(7)\cos 82^\circ$$

$$\Rightarrow c^2 = 144 + 49 - 168\cos 82^\circ$$

$$\therefore c = |ab| = 13 \text{ cm}$$