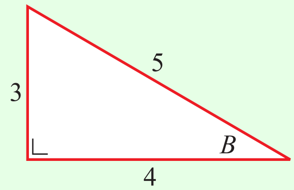
TRIGONOMETRY (Q 5, PAPER 2)

2003

5 (a) The lengths of the sides of a right-angled triangle are shown in the diagram and B is the angle indicated. Find the value of sin Bcos B, as a fraction.

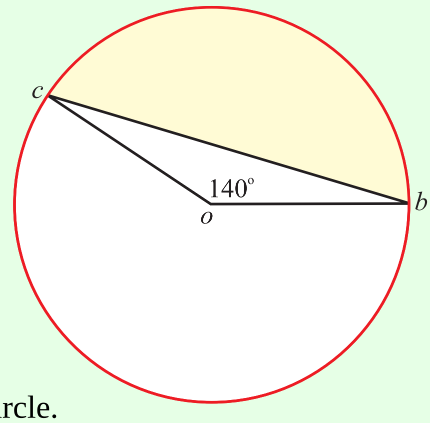


(b) A circle has centre o and radius 7 cm. The two points b and c are on the circle and $|\angle boc| = 140^\circ$.

(i) Find the area of the triangle obc, correct to the nearest cm^2.

(ii) Find the area of the sector obc, correct to the nearest cm^2.

(iii) Taking the areas correct to the nearest cm^2, express the area of the shaded region as a fraction of the total area enclosed by the circle. Give your answer as a fraction in its simplest form.



(c) One side of a triangle has length 8 cm and another has length 3 cm. The angle between these two sides measures 60°.

(i) Find the length of the third side.

(ii) Find the measures of the two remaining angles, correct to the nearest degree.

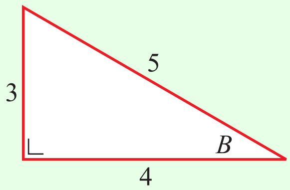
SOLUTION

5 (a)

$\sin B = \frac{3}{5}$

$\cos B = \frac{4}{5}$

$\therefore \sin B \cos B = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$



$\boxed{\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}}$ **3**

$\boxed{\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}}$ **4**

5 (b) (i)

AREA OF A NON RIGHT-ANGLED TRIANGLE

$\boxed{A = \frac{1}{2} ab \sin C}$ **6**

REMEMBER IT AS:

$\boxed{\text{Area} = \frac{1}{2} \times \text{Product of 2 sides} \times \text{Sine of the included angle}}$

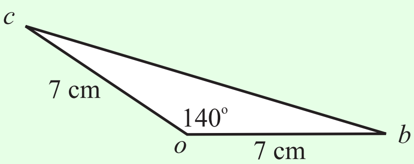
Area of triangle obc:

Area $= \frac{1}{2} \times$ Product of 2 sides $\times$ Sine of the included angle

$\Rightarrow A = \frac{1}{2}(7)(7)\sin 140^\circ$

$\Rightarrow A = \frac{49}{2}\sin 140^\circ$

$\therefore A = 16 \text{ cm}^2$



5 (b) (ii)

Area of sector *obc*: $A = \pi r^2 \times \dfrac{\theta}{360^\circ} \Rightarrow A = \pi (7)^2 \times \dfrac{140^\circ}{360^\circ}$

$\therefore A = 60 \text{ cm}^2$

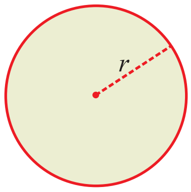
Area of sector

$\boxed{A = \pi r^2 \times \dfrac{\theta}{360^\circ}}$ **8**

5 (b) (iii)

Shaded area = Area of sector *obc* – Area of triangle *obc* = 60 – 16 = 44 cm^2

6. **Circle**



L: Length of Circumference

r: Radius

$\boxed{L = 2\pi r}$ **7**

$\boxed{A = \pi r^2}$ **8**

Area of circle: $A = \pi r^2 \Rightarrow A = \pi (7)^2 = 49\pi$

$\therefore A = 154 \text{ cm}^2$

$$\frac{\text{Area of shaded region}}{\text{Area of circle}} = \frac{44}{154} = \frac{2}{7}$$

5 (c) (i)

The Cosine Rule

$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ **10**

You use the Cosine rule when you are given:

[A] Two sides and one included angle,

[B] Three sides.

There are two other versions of the cosine rule not given in the tables:

$$\boxed{\begin{aligned} b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$ **10**

$a^2 = b^2 + c^2 - 2bc\cos A$

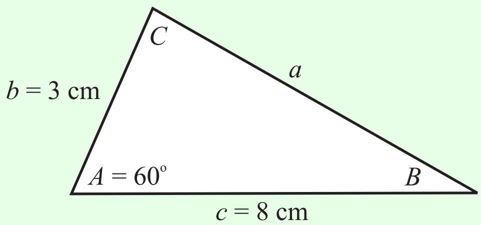
$\Rightarrow a^2 = 3^3 + 8^2 - 2(3)(8)\cos 60^\circ$

$\Rightarrow a^2 = 9 + 64 - 48(\tfrac{1}{2})$

$\Rightarrow a^2 = 9 + 64 - 24$

$\Rightarrow a^2 = 49$

$\therefore a = \sqrt{49} = 7 \text{ cm}$

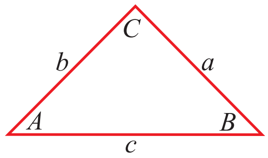


5 (c) (ii)

SINE RULE FORMULA

$$\boxed{\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}}\ \ldots\ldots\ 9 \qquad OR \qquad \boxed{\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}}\ \ldots\ldots\ 9$$

You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



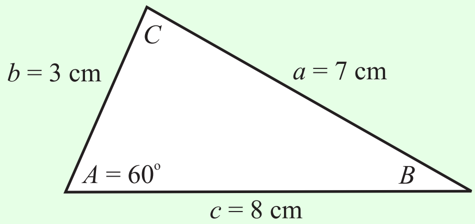
REMEMBER IT AS:

$$\boxed{\frac{\sin(\text{Angle 1})}{\text{Opposite side}}=\frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad OR \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})}=\frac{\text{Opposite side}}{\sin(\text{Angle 2})}}$$

$$\frac{\sin B}{b}=\frac{\sin A}{a} \Rightarrow \frac{\sin B}{3}=\frac{\sin 60^\circ}{7}$$

$$\Rightarrow \sin B=\frac{3\sin 60^\circ}{7}=0.3712$$

$$\therefore B=\sin^{-1}(0.3712)=22^\circ$$



The 3 angles in a triangle add up to 180°.

$$60^\circ+22^\circ+C=180^\circ \Rightarrow C=180^\circ-60^\circ-22^\circ$$

$$\therefore C=98^\circ$$