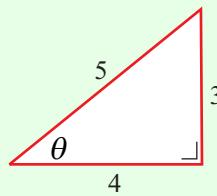


TRIGONOMETRY (Q 5, PAPER 2)**2002**

- 5 (a) Use the information given in the diagram to show that

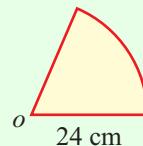
$$\sin \theta + \cos \theta > \tan \theta.$$



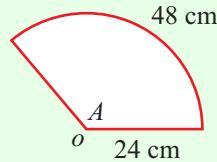
- (b) A circle has radius 24 cm and centre o .

- (i) Calculate the area of a sector which has 70° at o .

Take $\pi = \frac{22}{7}$.



- (ii) An arc of length 48 cm subtends an angle A at o . Calculate A , correct to the nearest degree.



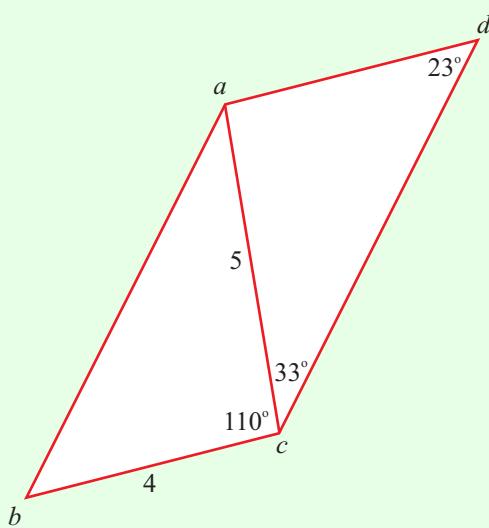
- (c) In the quadrilateral $abcd$, $|ac| = 5$ units,

$$|bc| = 4 \text{ units}, |\angle bca| = 110^\circ, |\angle acd| = 33^\circ$$

$$\text{and } |\angle cda| = 23^\circ.$$

- (i) Calculate $|ab|$, correct to two decimal places.

- (ii) Calculate $|cd|$, correct to two decimal places.

**SOLUTION****5 (a)**

$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = \frac{3}{4}$$

$$\sin \theta + \cos \theta = \frac{3}{5} + \frac{4}{5} = \frac{7}{5} > \frac{3}{4}$$

$$\therefore \sin \theta + \cos \theta > \tan \theta$$

$$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \dots\dots \text{3}$$

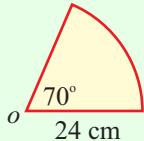
$$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}} \dots\dots \text{4}$$

$$\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}} \dots\dots \text{5}$$

5 (b) (i)

$$A = \pi r^2 \times \frac{\theta}{360^\circ} \Rightarrow A = \left(\frac{22}{7}\right)(24)^2 \left(\frac{70^\circ}{360^\circ}\right)$$

$$\therefore A = 352 \text{ cm}^2$$



Area of sector

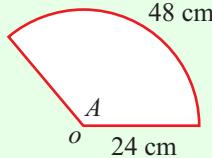
$$A = \pi r^2 \times \frac{\theta}{360^\circ} \dots\dots \boxed{8}$$

5 (b) (ii)

$$s = 2\pi r \times \frac{\theta}{360^\circ} \Rightarrow 48 = 2\pi(24) \left(\frac{A}{360^\circ}\right)$$

$$\Rightarrow 48 = 48\pi \left(\frac{A}{360^\circ}\right)$$

$$\therefore A = \frac{360}{\pi} = 115^\circ$$



Length of arc

$$s = 2\pi r \times \frac{\theta}{360^\circ} \dots\dots \boxed{7}$$

5 (c) (i)Consider triangle abc .**THE COSINE RULE**

$$a^2 = b^2 + c^2 - 2bc \cos A \dots\dots \boxed{10}$$

You use the Cosine rule when you are given:

- [A] Two sides and one included angle,
- [B] Three sides.

There are two other versions of the cosine rule not given in the tables:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

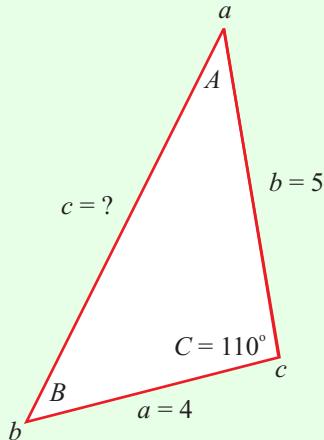
$$c^2 = a^2 + b^2 - 2ab \cos C \dots\dots \boxed{10}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\Rightarrow c^2 = 4^2 + 5^2 - 2(4)(5) \cos 110^\circ$$

$$\Rightarrow c^2 = 16 + 25 - 40 \cos 110^\circ$$

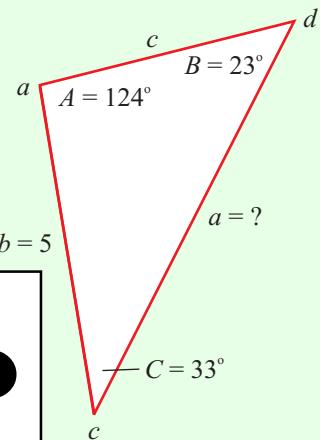
$$\therefore c = |ab| = 7.39 \text{ cm}$$



5 (c) (ii)Consider triangle adc .The 3 angles in a triangle add up to 180° .

$$A + 33^\circ + 23^\circ = 180^\circ$$

$$\therefore A = 124^\circ$$

**SINE RULE FORMULA**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

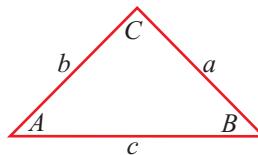
..... 9

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

..... 9

You use the Sine Rule when you are given:

- [A] Two angles and one side.
- [B] Two sides and one non-included angle.

**REMEMBER IT AS:**

$$\frac{\sin(\text{Angle 1})}{\text{Opposite side}} = \frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad \text{OR} \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})} = \frac{\text{Opposite side}}{\sin(\text{Angle 2})}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{a}{\sin 124^\circ} = \frac{5}{\sin 23^\circ}$$

$$\Rightarrow a = \frac{5 \sin 124^\circ}{\sin 23^\circ}$$

$$\therefore a = |cd| = 10.61 \text{ cm}$$