TRIGONOMETRY (Q 5, PAPER 2)

2001

5 (a) $\sin\theta = \frac{3}{5}$ where $0^\circ < \theta < 90^\circ$.

Find, without using the Tables or a calculator, the value of

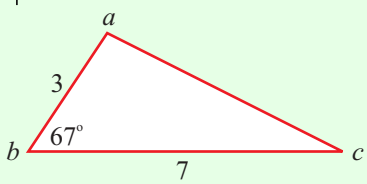
(i) $\cos\theta$

(ii) $\cos 2\theta$. [Note: $\cos 2\theta = \cos^2\theta - \sin^2\theta$.]

(b) In the triangle abc, $|ab| = 3$ units, $|bc| = 7$ units and $|\angle abc| = 67^\circ$.

(i) Calculate the area of the triangle abc, correct to one decimal place.

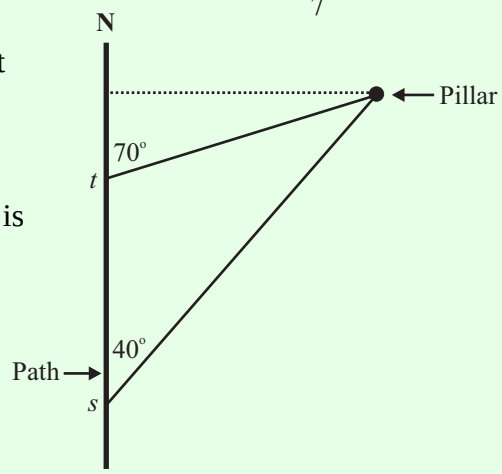
(ii) Calculate $|ac|$, correct to the nearest whole number.



(c) s and t are two points 300 m apart on a straight path due north.
From s the bearing of a pillar is N40°E.
From t the bearing of the pillar is N70°E.

(i) Show that the distance from t to the pillar is 386 m, correct to the nearest metre.

(ii) Find the shortest distance from the path to the pillar, correct to the nearest metre.

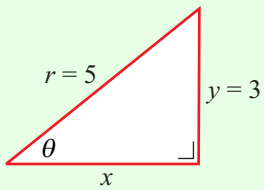


SOLUTION

5 (a) (i)

Draw a right-angled triangle.

$$\boxed{\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}} \quad \ldots\ldots\ 4$$



$$\sin\theta = \frac{3}{5} = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

Use Pythagoras to find x. $\boxed{x^2 + y^2 = r^2}$ 1

$x^2 + y^2 = r^2 \Rightarrow x^2 + 3^2 = 5^2$

$\Rightarrow x^2 + 9 = 25$

$\Rightarrow x^2 = 16$

$\therefore x = \sqrt{16} = 4$

$$\boxed{\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}} \quad \ldots\ldots\ 3$$

$\therefore \cos\theta = \frac{4}{5}$

5 (a) (ii)

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow \cos 2\theta = (\frac{4}{5})^2 - (\frac{3}{5})^2 = \frac{16}{25} - \frac{9}{25}$

$\therefore \cos 2\theta = \frac{7}{25}$

5 (b) (i)

AREA OF A NON RIGHT-ANGLED TRIANGLE

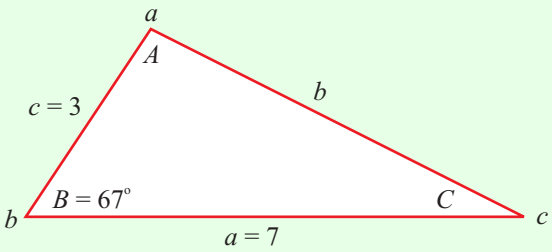
$\boxed{A = \frac{1}{2}ab\sin C}$ **6**

REMEMBER IT AS:

$\boxed{\text{Area} = \frac{1}{2} \times \text{Product of 2 sides} \times \text{Sine of the included angle}}$

Area $A = \frac{1}{2}ac\sin B = \frac{1}{2}(7)(3)\sin 67^o$

$\therefore A = 9.7$ square units



5 (b) (ii)

THE COSINE RULE

$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ **10**

You use the Cosine rule when you are given:
[A] Two sides and one included angle,
[B] Three sides.

There are two other versions of the cosine rule not given in the tables:

$$\boxed{\begin{aligned} b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}} \text{ ..... } \textbf{10}$$

$b^2 = a^2 + c^2 - 2ac\cos B$

$\Rightarrow b^2 = 7^2 + 3^2 - 2(7)(3)\cos 67^o$

$\Rightarrow b^2 = 49 + 9 - 42\cos 67^o$

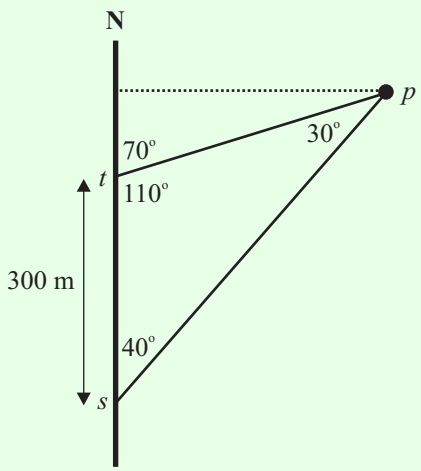
$\therefore b = |ac| = 6$

5 (c) (i)

Call the pillar p.

$|\angle pts| = 110^o$ [Straight angle]

$|\angle tps| = 30^o$ [3 angles of a triangle add up to 180°]

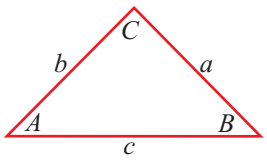


Consider triangle *pts*:

SINE RULE FORMULA

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{.....} \; 9 \quad \text{OR} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{.....} \; 9$$

You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



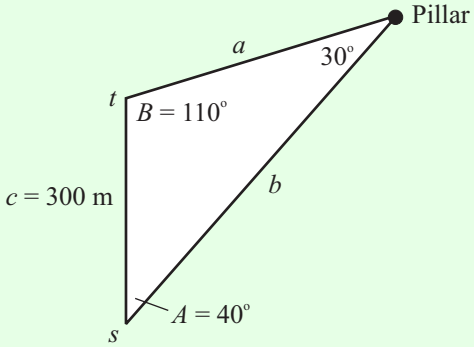
REMEMBER IT AS:

$$\frac{\sin(\text{Angle 1})}{\text{Opposite side}} = \frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad \text{OR} \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})} = \frac{\text{Opposite side}}{\sin(\text{Angle 2})}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{a}{\sin 40^\circ} = \frac{300}{\sin 30^\circ}$$

$$\Rightarrow a = \frac{300 \sin 40^\circ}{\sin 30^\circ}$$

$$\therefore a = |pt| = 386 \text{ m}$$



5 (c) (ii)

$$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \text{.....} \; 4$$

$$\sin 70^\circ = \frac{y}{386} \Rightarrow y = 386 \sin 70^\circ$$

$\therefore$ Shortest distance $y = 363$ m