## Trigonometry (Q 5, Paper 2)

1999

5 (a) $a b c$ is a right-angled triangle with $|\angle a c b|=90^{\circ}$, $|a b|=13,|b c|=5$ and $|a c|=12$.
Find, as fractions, the value of $\sin \angle a b c$ and the value of $\tan \angle b a c$.

(b) In the diagram, $o$ is the centre of the circle with radius length 5 and $p$ and $q$ are points on the circle. $|\angle p o q|=80^{\circ}$.
Find, correct to two places of decimals,
(i) the area of triangle poq
(ii) the area of the shaded region, taking $\pi=3 \cdot 14$.

(c) Two ships, $A$ and $B$, leave port $k$ at noon. $A$ is travelling due East and $B$ is travelling East $70^{\circ}$ South, as shown.
Calculate, to the nearest km, the distance between $A$ and $B$ when $A$ is 8 km from $k$ and $B$ is 12 km from $k$.

## Solution

5 (a)

$$
\sin A=\frac{y}{r}=\frac{\text { Opposite }}{\text { Hypotenuse }}
$$

$$
\sin \angle a b c=\frac{12}{13}
$$

$$
\tan A=\frac{y}{x}=\frac{\text { Opposite }}{\text { Adjacent }}
$$

5

$$
\tan \angle b a c=\frac{5}{12}
$$



## 5 (b) (i)

## Area of a non right-angled triangle

$$
A=\frac{1}{2} a b \sin C \ldots . . . .
$$

Remember it as:
Area $=\frac{1}{2} \times$ Product of 2 sides $\times$ Sine of the included angle

$A=\frac{1}{2}(5)(5) \sin 80^{\circ}=12.31$ square units

## 5 (b) (ii)

Area of shaded region = Area of sector poq - Area if triangle poq
Area of sector poq: $A=\pi r^{2} \times \frac{\theta}{360^{\circ}} \Rightarrow A=(3.14)(5)^{2} \times \frac{80^{\circ}}{360^{\circ}}$

> Area of sector $\therefore A=17.44$ square units

Area of shaded region $=17.44-12.31=5.13$ square units

## 5 (c)

The Cosine Rule

$$
a^{2}=b^{2}+c^{2}-2 b c \cos A
$$

10

You use the Cosine rule when you are given:
[A] Two sides and one included angle, [B] Three sides.

There are two other versions of the cosine rule not given in the tables:

$$
\begin{align*}
& b^{2}=a^{2}+c^{2}-2 a c \cos B \\
& c^{2}=a^{2}+b^{2}-2 a b \cos C \tag{10}
\end{align*}
$$

$a^{2}=b^{2}+c^{2}-2 b c \cos A$
$\Rightarrow a^{2}=12^{2}+8^{2}-2(12)(8) \cos 70^{\circ}$
$\Rightarrow a^{2}=144+64-192 \cos 70^{\circ}$
$\therefore a=12 \mathrm{~km}$


