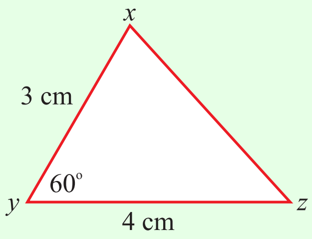
TRIGONOMETRY (Q 5, PAPER 2)

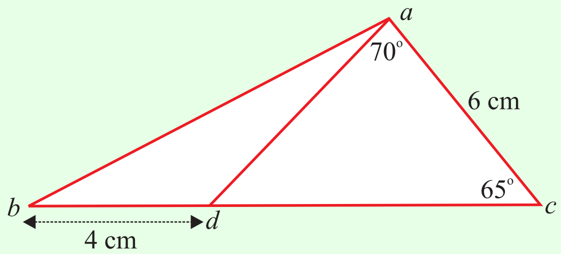
1997

5 (a) In the triangle xyz, $|xy| = 3$ cm, $|yz| = 4$ cm and $|\angle xyz| = 60^\circ$.
Use the cosine rule to find $|xz|$, correct to one place of decimals.
[See Tables, page 9.]



(b) θ is an acute angle where $\tan\theta = \frac{5}{12}$.
Find, as a fraction, the value of
(i) $\cos\theta$
(ii) $\sin\theta$
(iii) $\cos 2\theta$. [Note: $\cos 2\theta = \cos(\theta + \theta)$.]

(c) abc is a triangle and $d \in [bc]$, as shown.
If $|bd| = 4$ cm, $|ac| = 6$ cm, $|\angle acd| = 65^\circ$ and $|\angle dac| = 70^\circ$, find
(i) $|dc|$, correct to the nearest cm
(ii) the area of triangle abc, correct to the nearest cm^2.



SOLUTION

5 (a)

THE COSINE RULE

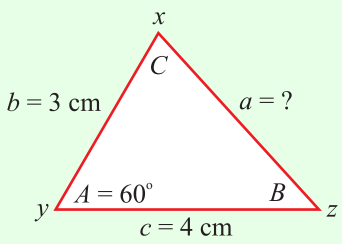
$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$ **10**

You use the Cosine rule when you are given:
[A] Two sides and one included angle,
[B] Three sides.

There are two other versions of the cosine rule not given in the tables:

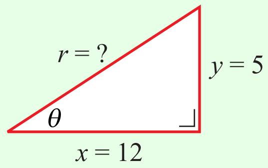
$$\boxed{\begin{array}{l} b^2 = a^2 + c^2 - 2ac\cos B \\ c^2 = a^2 + b^2 - 2ab\cos C \end{array}}$$ **10**

$b^2 = a^2 + c^2 - 2ac\cos B$
$a^2 = b^2 + c^2 - 2bc\cos A$
$\Rightarrow a^2 = 3^2 + 4^2 - 2(3)(4)\cos 60^\circ$
$\Rightarrow a^2 = 9 + 16 - 24(\frac{1}{2}) = 25 - 12$
$\therefore a = |xz| = \sqrt{13} = 3.6$ cm



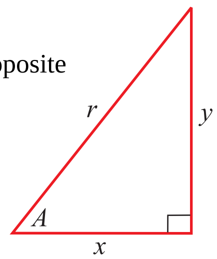
5 (b)

$\boxed{\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}}$ 5



Pythagoras

One of the angles in a right-angled triangle is 90°. The side opposite this angle is called the **hypotenuse**.
Pythagoras' theorem applies to right-angled triangles.



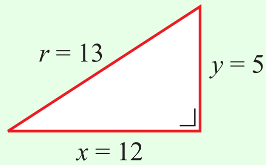
$\boxed{x^2 + y^2 = r^2}$ 1

$x^2 + y^2 = r^2 \Rightarrow 12^2 + 5^2 = r^2$

$\Rightarrow 144 + 25 = r^2$

$\Rightarrow r^2 = 169$

$\therefore r = \sqrt{169} = 13$



5 (b) (i)

$\cos\theta = \frac{12}{13}$

$\boxed{\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}}$ 3

5 (b) (ii)

$\sin\theta = \frac{5}{13}$

$\boxed{\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}}$ 4

5 (b) (iii)

$\boxed{\cos 2A = \cos^2 A - \sin^2 A}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = (\frac{12}{13})^2 - (\frac{5}{13})^2$

$\Rightarrow \cos 2\theta = \frac{144}{169} - \frac{25}{169}$

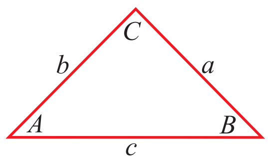
$\therefore \cos 2\theta = \frac{119}{169}$

5 (c) (i)

Sine Rule Formula

$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$ .... 9 *OR* $\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$ 9

You use the Sine Rule when you are given:
[A] Two angles and one side.
[B] Two sides and one non-included angle.



Remember it as:

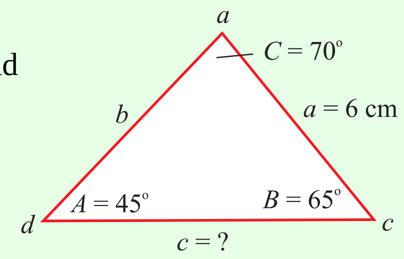
$\boxed{\frac{\text{sin(Angle 1)}}{\text{Opposite side}} = \frac{\text{sin(Angle 2)}}{\text{Opposite side}} \quad \textit{OR} \quad \frac{\text{Opposite side}}{\text{sin(Angle 1)}} = \frac{\text{Opposite side}}{\text{sin(Angle 2)}}}$

Consider triangle *adc*.
You can find angle A as the 3 angles in a triangle add up to 180°.

$$\frac{c}{\sin C}=\frac{a}{\sin A}\Rightarrow\frac{c}{\sin 70^\circ}=\frac{6}{\sin 45^\circ}$$

$$\Rightarrow c=\frac{6\sin 70^\circ}{\sin 45^\circ}$$

$$\therefore c=|dc|=8 \text{ cm}$$



5 (c) (ii)

AREA OF A NON RIGHT-ANGLED TRIANGLE

$\boxed{A=\tfrac{1}{2}ab\sin C}$ **6**

REMEMBER IT AS:

$\boxed{\text{Area }=\tfrac{1}{2}\times\text{Product of 2 sides }\times\text{Sine of the included angle}}$

$$A=\tfrac{1}{2}(6)(12)\sin 65^\circ$$

$$\therefore A=33 \text{ cm}^2$$