

TRIGONOMETRY (Q 5, PAPER 2)**1996**

- 5 (a) Find the length of an arc of a circle of radius length 6 cm subtending an angle of 120° at the centre. Give your answer in terms of π .

(b) A and B are acute angles where $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$.

Find, as fractions, the value of $\cos A$ and the value of $\sin B$.

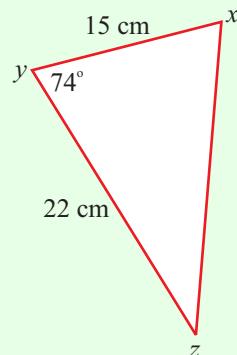
Find the value of $\sin(A + B)$, giving your answer as a single fraction.

- (c) xyz is a triangle where $|xy| = 15$ cm,
 $|yz| = 22$ cm and $|\angle xyz| = 74^\circ$.

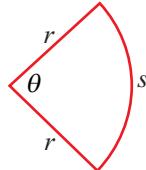
Find

- (i) $|xz|$, correct to the nearest cm

- (ii) $|\angle yxz|$, correct to the nearest degree.

**SOLUTION****5 (a)**

Length of arc: $s = 2\pi r \times \frac{\theta}{360^\circ}$ 7



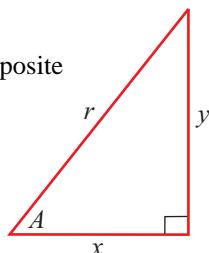
$$s = 2\pi r \times \frac{\theta}{360^\circ} = 2\pi(6) \times \frac{120^\circ}{360^\circ}$$

$$\therefore s = 4\pi \text{ cm}$$

5 (b)**PYTHAGORAS**

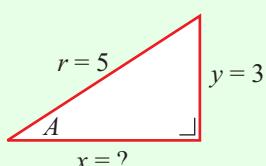
One of the angles in a right-angled triangle is 90° . The side opposite this angle is called the **hypotenuse**.

Pythagoras' theorem applies to right-angled triangles.



$$x^2 + y^2 = r^2$$
 1

$$\sin A = \frac{3}{5}$$



$$x^2 + y^2 = r^2 \Rightarrow x^2 + 3^2 = 5^2$$

$$\Rightarrow x^2 + 9 = 25$$

$$\Rightarrow x^2 = 16$$

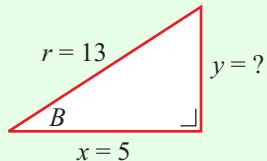
$$\therefore x = \sqrt{16} = 4$$

$$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$
 4

$$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$
 3

$$\therefore \cos A = \frac{4}{5}$$

$$\cos B = \frac{5}{13}$$



$$x^2 + y^2 = r^2 \Rightarrow 5^2 + y^2 = 13^2$$

$$\Rightarrow 25 + y^2 = 169$$

$$\Rightarrow y^2 = 144$$

$$\therefore y = \sqrt{144} = 12$$

$$\sin A = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

4

$$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

3

$$\therefore \sin B = \frac{12}{13}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\Rightarrow \sin(A+B) = \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$\Rightarrow \sin(A+B) = \frac{15}{65} + \frac{48}{65}$$

$$\therefore \sin(A+B) = \frac{63}{65}$$

COMPOUND ANGLES

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

5 (c) (i)

THE COSINE RULE

$$a^2 = b^2 + c^2 - 2bc \cos A$$

..... 10

You use the Cosine rule when you are given:

- [A] Two sides and one included angle,
- [B] Three sides.

There are two other versions of the cosine rule not given in the tables:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

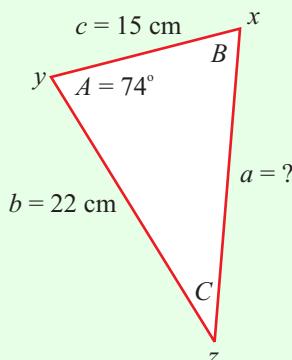
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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow a^2 = 22^2 + 15^2 - 2(22)(15) \cos 74^\circ$$

$$\Rightarrow a^2 = 484 + 225 - 660 \cos 74^\circ$$

$$\therefore a = |xz| = 23 \text{ cm}$$



5 (c) (ii)**SINE RULE FORMULA**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

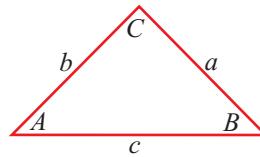
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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

..... **9**

You use the Sine Rule when you are given:

- [A] Two angles and one side.
 [B] Two sides and one non-included angle.

**REMEMBER IT AS:**

$$\frac{\sin(\text{Angle 1})}{\text{Opposite side}} = \frac{\sin(\text{Angle 2})}{\text{Opposite side}} \quad \text{OR} \quad \frac{\text{Opposite side}}{\sin(\text{Angle 1})} = \frac{\text{Opposite side}}{\sin(\text{Angle 2})}$$

$$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow \frac{\sin B}{22} = \frac{\sin 74^\circ}{23}$$

$$\Rightarrow \sin B = \frac{22 \sin 74^\circ}{23} = 0.9195$$

$$\therefore B = \sin^{-1}(0.9195) = 67^\circ$$

