Sequences & Series (Q 5, Paper 1)

Lesson No. 9: Proving a Sequence is Geometric

2002

5 (c) The first three terms of a geometric sequence are

$k-3,\ 2k-4,\ 4k-3,.....$

where k is a real number.

(i) Find the value of k.

(ii) Hence, write down the value of each of the first four terms of the sequence.

Solution

5 (c) (i)

As it is a geometric sequence, dividing any two consecutive terms gives you the same constant. This constant is the common ratio, r.

$\frac{(2k-4)}{(k-3)} = \frac{(4k-3)}{(2k-4)}$ [Multiply both sides by $(k-3)(2k-4)$ or cross multiply.]

$\Rightarrow (2k-4)(2k-4) = (4k-3)(k-3)$ [Multiply out the brackets.]

$\Rightarrow 4k^2 - 8k - 8k + 16 = 4k^2 - 12k - 3k + 9$

$\Rightarrow -16k + 16 = -15k + 9$

$\Rightarrow 16 - 9 = -15k + 16k$

$\Rightarrow 7 = k$

5 (c) (ii)

$k-3,\ 2k-4,\ 4k-3,...$ [Replace k by its value.]

$= (7)-3,\ 2(7)-4,\ 4(7)-3,...$

$= 7-3,\ 14-4,\ 28-3,...$

$= 4,\ 10,\ 25,...$

To get the fourth term, find the common ratio.

$\therefore r = \frac{10}{4} = \frac{5}{2}$

Multiply the third term by r.

$\therefore T_4 = \frac{5}{2} \times 25 = \frac{125}{2}$

The first four terms are: $4,\ 10,\ 25,\ \frac{125}{2}$