SEQUENCES & SERIES (Q 5, PAPER 1)

LESSON NO. 8: GEOMETRIC SERIES

2007			
5 (c) The first two terms of a geometric series are $\frac{1}{3} + \frac{1}{9} + \dots$			
(i) Find <i>r</i> , the common ratio.			
(ii) Find an expression for S_n , the sum of the first <i>n</i> terms.			
Write your answer in the form $\frac{1}{k}\left(1-\frac{1}{3^n}\right)$ where $k \in \mathbb{N}$.			
(iii) The sum of the first <i>n</i> terms of the geometric series $\frac{p}{3} + \frac{p}{9} + \dots$ is $1 - \frac{1}{3^n}$.			
Find the value of <i>p</i> .			
Solution 5 (c)			
Geometric series: $\frac{1}{3} + \frac{1}{9} + \dots$			
5 (c) (i)			
$r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{9} \times \frac{3}{1} = \frac{1}{3}$ Any term ÷ Previous term = $\frac{T_n}{T_{n-1}}$ = Constant (r)			
5 (c) (ii)			
$a = \frac{1}{3}$ $r = \frac{1}{3}$ Summing formula: $S_n = \frac{a(1-r^n)}{(1-r)}$ 5			
3			
$\therefore S_n = \frac{a(1-r^n)}{1-r} = \frac{\left(\frac{1}{3}\right)\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\left(\frac{1}{3}\right)}$			
$=\frac{(\frac{1}{3})(1-(\frac{1}{3})^n)}{\frac{2}{3}}$			
$=\frac{1}{2}\left(1-\frac{1}{3^n}\right)$			
5 (c) (iii)			
Geometric series: $\frac{p}{3} + \frac{p}{9} + \dots$ $a = \frac{p}{3}$			
$r = \frac{\frac{p}{9}}{\frac{p}{3}} = \frac{1}{3}$			
$S_n = \frac{a(1-r^n)}{1-r} = \frac{\left(\frac{p}{3}\right)\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$			
$=\frac{(\frac{p}{3})(1-(\frac{1}{3})^n)}{\frac{2}{3}}$			
$\overline{3} = \left(\frac{p}{2}\right)\left(1 - \left(\frac{1}{3}\right)^n\right)$			
$=\frac{p}{2}\left(1-\frac{1}{3^n}\right)$			
$=\frac{1}{2}\left(1-\frac{1}{3^{n}}\right)$ Cont.			

You are told that the sum of the first *n* terms is given by $1 - \frac{1}{3^n}$.

$$\therefore \frac{p}{2} \left(1 - \frac{1}{3^n} \right) = 1 - \frac{1}{3^n}$$
$$\Rightarrow \frac{p}{2} = 1$$
$$\Rightarrow p = 2$$

2006

5 (b) The *n*th term of a geometric series is

 $T_n = 4(\frac{1}{2})^n.$

(i) Find *a*, the first term.

(ii) Find *r*, the common ratio.

(iii) Write $4 - S_{10}$ in the form $\frac{1}{2^k}$, $k \in \mathbb{N}$, where S_{10} is the sum of the first ten terms. Solution

5 (b) (i)

Replace n by 1 in the general term to find a.

$$a = T_1$$

$$T_n = 4(\frac{1}{2})^n$$

$$\Rightarrow T_1 = 4(\frac{1}{2})^1 = 4(\frac{1}{2}) = 2$$

5 (b) (ii)

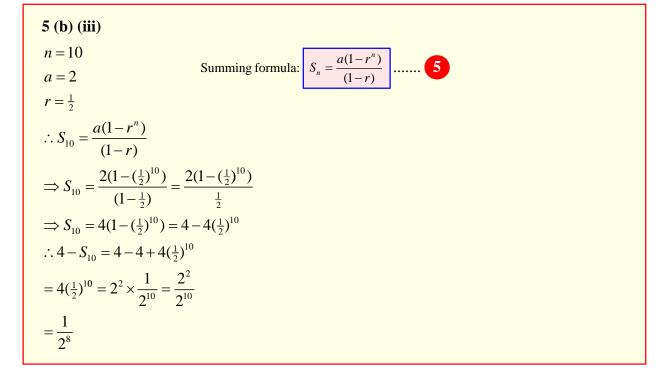
To find the common ratio, r, find the second term, T_2 , and then divide the second term by the first term.

$$T_{n} = 4(\frac{1}{2})^{n}$$

$$\Rightarrow T_{2} = 4(\frac{1}{2})^{2} = 4(\frac{1}{4}) = 1$$

$$\therefore r = \frac{T_{2}}{T_{1}} = \frac{1}{2}$$

CONT...



5 (c) The first term of a geometric series is 1 and the common ratio is -4.(i) Write down the first three terms of the series.

(ii) Find S_6 , the sum of the first 6 terms.

(iii) Show that $16S_4 - 3 = S_6$, where S_4 is the sum of the first 4 terms.

Summing formula: $S_n = \frac{a(1-r^n)}{(1-r)}$ 5

SOLUTION

5 (c) (i)

Write down the first term and keep on multiplying by the common ratio.

$$a = 1, r = -4$$

Geometric series: 1, -4, 16, ...

5 (c) (ii)

$$n = 6, a = 1, r = -4$$

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

$$\Rightarrow S_6 = \frac{1(1 - (-4)^6)}{(1 - (-4))}$$

$$\Rightarrow S_6 = \frac{(1 - 4096)}{5}$$

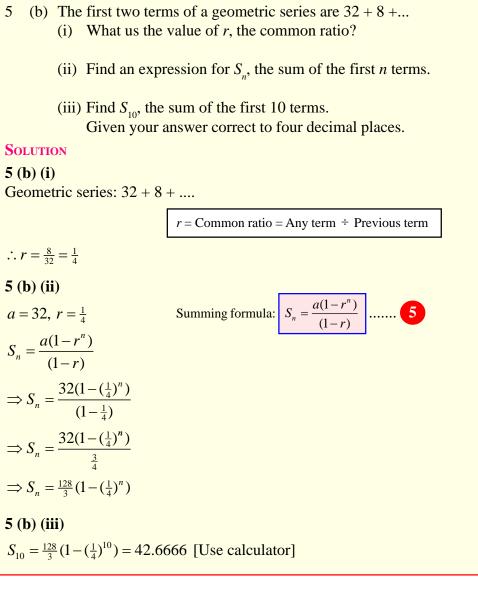
$$\Rightarrow S_6 = \frac{-4095}{5} = -816$$

CONT...

5 (c) (iii)

$$n = 4, a = 1, r = -4$$

 $16S_4 - 3 = 16\left(\frac{1(1 - (-4)^4)}{(1 - (-4))}\right) - 3$
 $= 16\left(\frac{(1 - 256)}{5}\right) - 3$
 $= 16\left(\frac{-255}{5}\right) - 3$
 $= 16(-51) - 3$
 $= -816 - 3 = -819$



2001 5 (b) The *n*th term of a geometric series is given by $T_n = 3^n$. (i) What is the value of *a*, the first term? (ii) What is the value of r, the common ratio? (iii) Show that S_{10} , the sum of the first ten terms, is $\frac{3}{2}(3^{10}-1)$. **SOLUTION** 5 (b) (i) $T_n = 3^n$ $\therefore T_1 = a = 3^1 = 3$ 5 (b) (ii) r =Common ratio = Any term \div Previous term Find the second term, T_2 , and then find the common ratio r by dividing the second term by the first term. $T_2 = 3^2 = 9$ ∴ $r = \frac{9}{3} = 3$ 5 (b) (iii) a = 3, r = 3, n = 10Summing formula: $S_n = \frac{a(1-r^n)}{(1-r)}$ 5 $S_n = \frac{a(1-r^n)}{(1-r)}$ $\Rightarrow S_{10} = \frac{3(1-3^{10})}{(1-3)}$ $\Rightarrow S_{10} = \frac{3(1-3^{10})}{-2}$ $\Rightarrow S_{10} = -\frac{3}{2}(1-3^{10})$ $\Rightarrow S_{10} = \frac{3}{2}(3^{10}-1)$

- 5 (b) The first term of a geometric series is 1 and the common ratio is $\frac{11}{10}$.
 - (i) Write down the second, thirds and fourth terms of the series.
 - (ii) Calculate S_4 , the sum of the first four terms. Give your answer as a decimal.

SOLUTION

5 (b) (i)

To generate the terms of a geometric series multiply each term by the ratio to get the next term.

 $a = T_1 = 1$ $T_2 = 1 \times \frac{11}{10} = \frac{11}{10}$ $T_3 = \frac{11}{10} \times \frac{11}{10} = \frac{121}{100}$ $T_4 = \frac{121}{100} \times \frac{11}{10} = \frac{1331}{1000}$ 5 (b) (ii) $S_4 = T_1 + T_2 + T_3 + T_4 = 1 + \frac{11}{10} + \frac{121}{100} + \frac{1331}{1000} = 1 + 1.1 + 1.21 + 1.331 = 1.641$

1999

- 5 (b) The first two terms of a geometric series are $2 + \frac{2}{3} + \dots$
 - (i) Find *r*, the common ratio.
 - (ii) Write down the third and fourth terms of the series.

(iii) Show that S_6 , the sum to 6 terms, is $3 - \frac{1}{3^5}$.

SOLUTION 5 (b) (i)

r-

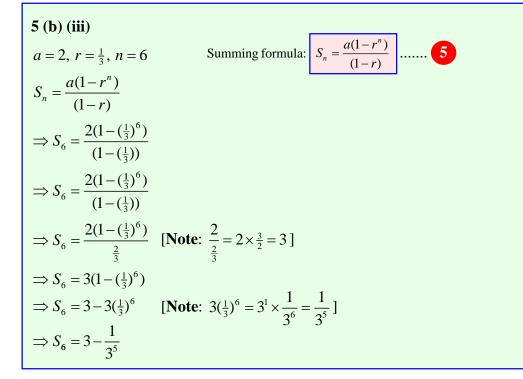
r =Common ratio = Any term \div Previous term

$$r = \frac{\frac{2}{3}}{2} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

5 (b) (ii)

To generate the terms of a geometric sequence, keep on multiplying each term by the common ratio r to get the next term.

$$T_3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$
$$T_4 = \frac{2}{9} \times \frac{1}{3} = \frac{2}{27}$$
Cont...



5 (b) The *n*th term of a geometric sequence is

$$T_n = \frac{2^n}{3^n}.$$

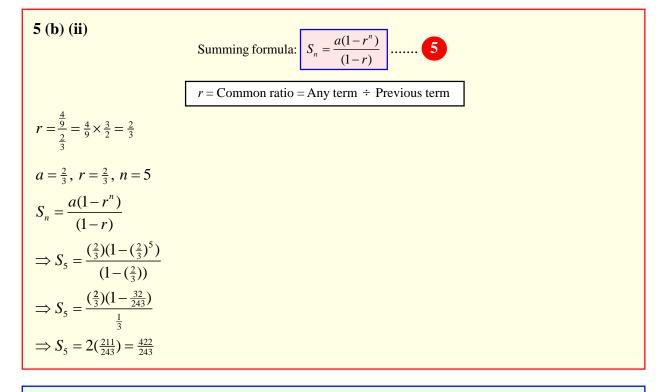
(i) Find the first three terms of the sequence.

(ii) Show that S_5 , the sum of the first five terms, is $\frac{422}{243}$.

SOLUTION

5 (b) (i) $T_n = \frac{2^n}{3^n}$ $\therefore T_1 = \frac{2^{(1)}}{3^{(1)}} = \frac{2}{3}$ $\therefore T_2 = \frac{2^{(2)}}{3^{(2)}} = \frac{4}{9}$ $\therefore T_3 = \frac{2^{(3)}}{3^{(3)}} = \frac{8}{27}$ Geometric sequence: $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \dots$

CONT...



5 (b) The *n*th term, T_n , of a geometric series is

 $T_n = 3^{n-1}.$ Find (i) T_1 , the first term

(ii) r, the common ratio

(iii) S_n , the sum to *n* terms. Investigate if

$$2S_n - T_n = 2T_n - 1.$$

SOLUTION

5 (b) (i) $T_n = 3^{n-1} \Longrightarrow T_1 = 3^{(1)-1} = 3^0 = 1$ **5 (b) (ii)**

r =Common ratio = Any term ÷ Previous term

Find the second term T_2 and then divide the second term by the first term to find r.

$$T_n = 3^{n-1} \Longrightarrow T_2 = 3^{(2)-1} = 3^1 = 3$$
$$\Longrightarrow r = \frac{T_2}{T_1} = \frac{3}{1} = 3$$

Сомт...

5 (b) (iii) $a(1-r^n)$		
$a = 1, r = 3$ Summing formula: $S_n = \frac{a(1-r^n)}{(1-r)}$ 5		
$S_n = \frac{a(1-r^n)}{(1-r)}$		
$\implies S_n = \frac{1(1-3^n)}{(1-3)}$		
$\Rightarrow S_n = \frac{1(1-3^n)}{-2}$		
$\Rightarrow S_n = \frac{1}{2}(3^n - 1)$		
LHS	RHS	
$2S_n - T_n$	$2T_n - 1$	
$= 2[\frac{1}{2}(3^{n}-1)] - 3^{n-1}$	$=2(3^{n-1})-1$	
$=3^{n}-1-3^{n-1}$		
$=(3^n-3^{n-1})-1$		
$=3^{n-1}(3^1-1)-1$		
$=3^{n-1}(2)-1$		
$\therefore 2S_n - T_n = 2T_n - 1$		