Sequences & Series (Q 5, Paper 1)

Lesson No. 7: Geometric Sequences

2005

5 (c) In a geometric sequence of positive terms, the third term is $\frac{1}{4}$ and the fifth term is $\frac{1}{16}$.

(i) Find r, the common ratio.

(ii) Find a, the first term.

(iii) How many terms of the sequence are greater than 0·01?

Solution

5 (c)

General term: $T_n = ar^{n-1}$ 4

Ex. The tenth term of a geometric sequence: $T_{10} = ar^9$

5 (c) (i)

$T_3 = ar^{n-1} = ar^2 = \frac{1}{4}$

$T_5 = ar^{n-1} = ar^4 = \frac{1}{16}$

Dividing $\Rightarrow \frac{ar^4}{ar^2} = \frac{\frac{1}{16}}{\frac{1}{4}} \Rightarrow r^2 = \frac{1}{16} \times \frac{4}{1} = \frac{1}{4} \Rightarrow r = \pm\frac{1}{2}$

As it is a geometric sequence of positive terms take $r = \frac{1}{2}$.

5 (c) (ii)

$ar^2 = \frac{1}{4} \Rightarrow a(\frac{1}{2})^2 = \frac{1}{4}$

$\Rightarrow a(\frac{1}{4}) = \frac{1}{4} \Rightarrow a = 1$

5 (c) (iii)

How many terms are greater that $0.01 = \frac{1}{100}$?

Write them out and count the number of the terms that are greater than 0.01.

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \ldots$

You can see that each of the first **seven** terms are greater than 0.01.

2003

5 (a) The first term of a geometric sequence is 4 and the commom ratio is 1·5.
Write down the next three terms of the sequence.

Solution

Write down the first term and keep on multiplying by the common ratio, r, to generate the terms of the geometric sequence.

$4, 6, \frac{27}{2}, \ldots$

1997

5 (a) $T_1 + T_2 + T_3 + \ldots$ is a geometric series.
The first term, T_1, is 1 and the common ratio is 2.
Show that
$T_3 + T_5 = 2(T_2 + T_4)$.

SOLUTION

> To produce a **GEOMETRIC SEQUENCE**, start with a number, *a*, and keep on multiplying by a number, *r*, forever.

Geometric series: $1 + 2 + 4 + 8 + 16 + \ldots$

$T_1 = 1$, $T_2 = 2$, $T_3 = 4$, $T_4 = 8$, $T_5 = 16$

$T_3 + T_5 = 4 + 16 = 20$

$2(T_2 + T_4) = 2(2 + 8) = 2(10) = 20$

$\therefore T_3 + T_5 = 2(T_2 + T_4)$