SEQUENCES & SERIES (Q 5, PAPER 1)

LESSON NO. 6: PROVING A SEQUENCE IS ARITHMETIC

2006

5 (c) The first three terms of an arithmetic sequence are

$h+3, 5h-2, 6h-13$

where h is a real number.

(i) Find the value of h.

(ii) Hence, write down the value of each of the first three terms.

(iii) Find the value of the eleventh term.

SOLUTION

5 (c) $h+3, 5h-2, 6h-13$

5 (c) (i)

> **TEST**: Any term − Previous term = $T_n - T_{n-1}$ = Constant (d)

As it is an arithmetic sequence, subtracting any two consecutive terms gives you the same constant. This constant is the common difference, d.

$5h-2-(h+3) = 6h-13-(5h-2)$

$\Rightarrow 5h-2-h-3 = 6h-13-5h+2$

$\Rightarrow 4h-5 = h-11$

$\Rightarrow 4h-h = -11+5$

$\Rightarrow 3h = -6$

$\Rightarrow h = -2$

5 (c) (ii)

$h+3, 5h-2, 6h-13$

$= (-2)+3, 5(-2)-2, 6(-2)-13$

$= -2+3, -10-2, -12-13$

$= 1, -12, -25$

5 (c) (ii)

$a = 1,$

$d = 12-1 = -13$

$n = 11$

$T_n = a+(n-1)d$

$\Rightarrow T_n = 1+(11-1)(-13)$

$\Rightarrow T_n = 1+(10)(-13)$

$\Rightarrow T_n = 1-130 = -129$

General term: $\boxed{T_n = ar^{n-1}}$ **4**

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5   (c)   The nth term of a series is given by
$$T_n = 4n + 1.$$

(i)   Write down, in terms of n, an expression for T_{n-1}, the $(n-1)$st. term.

(ii)   Show that the series is arithmetic.

(iii)   Find S_{20}, the sum of the first 20 terms of the series.

SOLUTION

5 (c) (i)

Replace n by $(n-1)$.

$T_n = 4n + 1$

$\Rightarrow T_{n-1} = 4(n-1) + 1$

$\Rightarrow T_{n-1} = 4n - 4 + 1$

$\Rightarrow T_{n-1} = 4n - 3$

5 (c) (ii)

> **TEST THAT A SERIES IS ARITHMETIC**: Any term – Previous term $= T_n - T_{n-1} = $ Constant (d)

$T_n - T_{n-1} = 4n + 1 - (4n - 3)$

$= 4n + 1 - 4n + 3$

$= 4$

Therefore, the series is arithmetic because 4 is a constant. This constant is the common difference d.

5 (c) (ii)

$a = 1,$ General term: $\boxed{T_n = ar^{n-1}}$ **4**

$d = 12 - 1 = -13$

$n = 11$

$T_n = a + (n-1)d$

$\Rightarrow T_n = 1 + (11-1)(-13)$

$\Rightarrow T_n = 1 + (10)(-13)$

$\Rightarrow T_n = 1 - 130 = -129$