# SEQUENCES & SERIES (Q 5, PAPER 1)

## LESSON NO. 5: ARITHMETIC SERIES

**2007**

5 (b) The first term of an arithmetic series is 3 and the common difference is 4.

(i) Find, in terms of $n$, an expression for $T_n$, the $n$th term.

(ii) How many terms of the series are less than 200?

(iii) Find the sum of these terms.

**SOLUTION**

**5 (b)**

Arithmetic series

$a = 3$

$d = 4$

General term: $T_n = a + (n-1)d$ ..... 2

**5 (b) (i)**

$T_n = a + (n-1)d = 3 + (n-1)4$

$= 3 + 4n - 4$

$= 4n - 1$

**5 (b) (ii)**

You are being asked to solve $T_n < 200$ for $n$.

$T_n < 200 \Rightarrow 4n - 1 < 200$

$\Rightarrow 4n < 200 + 1$

$\Rightarrow 4n < 201$

$\Rightarrow n < \frac{201}{4}$

$\Rightarrow n < 50.25$

$n$ must be a whole positive number. Therefore, $n = 50$.

**5 (b) (iii)**

Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ ..... 3

You are asked to find the sum of 50 terms ($n = 50$) where $a = 3$ and $d = 4$.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{50} = \frac{50}{2}[2(3) + (50-1)4]$

$\Rightarrow S_{50} = 25[6 + 49(4)]$

$\Rightarrow S_{50} = 25[6 + 196]$

$\Rightarrow S_{50} = 25[202]$

$\Rightarrow S_{50} = 5050$

**2005**

5 (b) The sum of the first $n$ terms of an arithmetic series is given by

$S_n = n^2 + n.$

(i) Find $a$, the first term.

(ii) Find $S_2$, the sum of the first two terms.

(iii) Find $d$, the common difference.

(iv) Write down the first five terms of the series.

**SOLUTION**

**5 (b)**

$S_n = n^2 + n$

**5 (b) (i)**

$S_1 = T_1 = a$

$\Rightarrow S_1 = (1)^2 + (1) = 1 + 1 = 2$

$S_1 = T_1$ for all sequences and series.

**5 (b) (ii)**

$S_2 = (2)^2 + (2) = 4 + 2 = 6$

**5 (b) (iii)**

$S_n - S_{n-1} = T_n \Rightarrow S_2 - S_1 = T_2$

$\Rightarrow T_2 = 6 - 2 = 4$

$S_n - S_{n-1} = T_n$ ..... 1

The first two terms of an arithmetic sequence are: 2, 4,...

$d$ = Common difference = Any term − Previous term

$\therefore d = 4 - 2 = 2$

**5 (b) (iv)**

Keep on adding the common difference, 2, to each term to get the next term.

The first five terms of the arithmetic sequence are 2, 4, 6, 8, 10.

**2004**

5 (b) The *n*th term of an arithmetic series is given by

$T_n = 1 + 5n$.

(i) The first term is $a$ and the common difference is $d$.
Find the value of $a$ and the value of $d$.

(ii) Find the value of $n$ for which $T_n = 156$.

(iii) Find $S_{12}$, the sum of the first 12 terms.

**SOLUTION**

**5 (b) (i)**

Generate the first 2 terms of the arithmetic sequence by letting $n = 1$ and then letting $n = 2$.

$T_n = 1 + 5n$

$\Rightarrow T_1 = 1 + 5(1) = 1 + 5 = 6$

$\Rightarrow T_2 = 1 + 5(2) = 1 + 10 = 11$

Arithmetic sequence: 6, 11,....

$d$ = Common difference = Any term − Previous term

First term $a = 6$

Common difference $d = 11 - 6 = 5$

**5 (b) (ii)**

$T_n = 156 \Rightarrow 1 + 5n = 156$

$\Rightarrow 5n = 156 - 1$

$\Rightarrow 5n = 155$

$\Rightarrow n = \frac{155}{5}$

$\Rightarrow n = 31$

**5 (b) (iii)**

Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ ..... 3

$a = 6$, $d = 5$, $n = 12$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{12} = \frac{12}{2}[2(6) + (12-1)(5)]$

$\Rightarrow S_{12} = 6[12 + (11)(5)]$

$\Rightarrow S_{12} = 6[12 + 55]$

$\Rightarrow S_{12} = 6[67]$

$\Rightarrow S_{12} = 402$

**2003**

5 (c) The fifth term of an arithmetic series is 21 and the tenth term is 11.

(i) Find the first term and the common difference.

(ii) Find the sum of the first twenty terms.

(iii) For what value of $n > 0$ is the sum of the first $n$ terms equal to zero?

**SOLUTION**

**5 (c) (i)**

General term: $T_n = a + (n-1)d$ ..... 2

**Ex**. The fifty-sixth term of an arithmetic sequence: $T_{56} = a + 55d$

$T_5 = a + 4d = 21 \ldots (1)$

$T_{10} = a + 9d = 11 \ldots (2)$ ← Solve simultaneously by subtracting.

$-5d = 10 \Rightarrow d = -2$

Substitute this value of $d$ back into Eqn. (**1**): $a + 4(-2) = 21 \Rightarrow a - 8 = 21 \Rightarrow a = 29$

**5 (c) (ii)**

Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ ..... 3

$a = 29,\ d = -2,\ n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_n = \frac{10}{2}[2(29) + (10-1)(-2)]$

$\Rightarrow S_n = 5[58 + (9)(-2)]$

$\Rightarrow S_n = 5[58 - 18]$

$\Rightarrow S_n = 5[40] = 200$

**5 (c) (iii)**

Put $S_n = 0$ and solve for $n$.

$a = 29,\ d = -2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_n = \frac{n}{2}[2(29) + (n-1)(-2)] = 0$

$\Rightarrow \frac{n}{2}[58 + (n-1)(-2)] = 0$

$\Rightarrow \frac{n}{2}[58 - 2n + 2] = 0$

$\Rightarrow \frac{n}{2}[60 - 2n] = 0$

$\Rightarrow n[30 - n] = 0$ [Set each factor equal to zero and solve for $n$.]

$\Rightarrow n = 0,\ 30$

As $n > 0$, the answer is $n = 30$.

**2001**

5 (c) The sum of the first n terms of an arithmetic series is given by

$$S_n = 4n^2 - 8n.$$

(i) Use $S_1$ and $S_2$ to find the first term and the common difference.

(ii) Starting with the first term, how many terms of the series must be added to give a sum of 252?

**SOLUTION**

**5 (c) (i)**

$S_n = 4n^2 - 8n$

$\therefore S_1 = 4(1)^2 - 8(1) = 4 \times 1 - 8 = 4 - 8 = -4$

$\therefore S_2 = 4(2)^2 - 8(2) = 4 \times 4 - 16 = 16 - 16 = 0$

$S_1 = T_1$ for all sequences and series.

$S_n - S_{n-1} = T_n$ ..... 1

$S_1 = a = -4$

$S_2 - S_1 = T_2 = 0 - (-4) = 0 + 4 = 4$

Arithmetic sequence: $-4, 4, \ldots$

$d = 4 - (-4) = 4 + 4 = 8$

**5 (c) (ii)**

Put $S_n$ equal to 252 and solve for $n$.

$S_n = 252$

$\Rightarrow 4n^2 - 8n = 252$

$\Rightarrow 4n^2 - 8n - 252 = 0$

$\Rightarrow n^2 - 2n - 63 = 0$

$\Rightarrow (n - 9)(n + 7) = 0$

$\therefore n = 9, -7$

Ignore the negative solution as $n$ must be positive.
Therefore, 9 terms must be added together to give a total of 252.

**2000**

5 (c) The first three terms of an arithmetic series are 5 + 10 + 15 + ....

(i) Find, in terms of $n$, an expression for $T_n$, the $n$th term.

(ii) Find, in terms of $n$, an expression for $S_n$, the sum to $n$ terms.

(iii) Using your expression for $S_n$, find the sum of the natural numbers that are both multiples of 5 and smaller than 1000.

**SOLUTION**

**5 (c) (i)**

$a = 5,\ d = 5$ General term: $T_n = a + (n-1)d$ ..... 2

$T_n = a + (n-1)d$

$\Rightarrow T_n = 5 + (n-1)(5)$

$\Rightarrow T_n = 5 + 5n - 5$

$\Rightarrow T_n = 5n$

**5 (c) (ii)**

$a = 5,\ d = 5$ Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ ..... 3

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_n = \frac{n}{2}[2(5) + (n-1)(5)]$

$\Rightarrow S_n = \frac{n}{2}[10 + 5n - 5]$

$\Rightarrow S_n = \frac{n}{2}[5n + 5]$

**5 (c) (iii)**

The series contains terms that are multiples of 5. Put $T_n$ equal to 1000 and solve for $n$. This will tell you the number of terms that are smaller than 1000. Now you know the number of terms you need to add together.

$T_n = 5n$

$\Rightarrow 5n = 1000$

$\Rightarrow n = 200$

The 200th. term is 1000. Therefore, 199 terms are less than 1000. Add together the first 199 terms.

$a = 5,\ d = 5,\ n = 199$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{199} = \frac{199}{2}[2(5) + (199-1)(5)]$

$\Rightarrow S_{199} = \frac{199}{2}[10 + (198)(5)]$

$\Rightarrow S_{199} = \frac{199}{2}[10 + 990]$

$\Rightarrow S_{199} = \frac{199}{2}[1000]$

$\Rightarrow S_{199} = 199[500] = 99,500$

**1998**

5 (c) The first three terms of an arithmetic series are

$2d + 3d + 4d + ...$

where $d$ is a real number.

(i) Find, in terms of $d$, an expression for $T_{10}$, the tenth term.

(ii) Find, in terms of $d$, an expression for $S_{10}$, the sum to 10 terms.

(iii) If $S_{10} - T_{10} = 162$, find the value of $d$ and write down the first four terms of the series.

**SOLUTION**

**5 (c) (i)**

General term: $T_n = a + (n-1)d$ ..... 2

Arithmetic sequence: $2d$, $3d$, $4d$,...

$d$ = Common difference = Any term − Previous term

Common difference $3d - 2d = d$

$a = 2d,\ d = d,\ n = 10$

$T_n = a + (n-1)d$

$\Rightarrow T_{10} = 2d + (10-1)d$

$\Rightarrow T_{10} = 2d + 9d = 11d$

**5 (c) (ii)**

Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ ..... 3

$a = 2d,\ d = d,\ n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2(2d) + (10-1)d]$

$\Rightarrow S_{10} = 5[4d + 9d]$

$\Rightarrow S_{10} = 5[13d] = 65d$

**5 (c) (iii)**

$S_{10} - T_{10} = 162$

$\Rightarrow 65d - 11d = 162$

$\Rightarrow 54d = 162$

$\Rightarrow d = \frac{162}{54} = 3$

Replace $d$ by 3 to generate the series:

Arithmetic series: 6 + 9 + 12 + 15

**1996**

5 (a) The first two terms of an arithmetic series are given as
2 + 8 +.....
Find

(i) $d$, the common difference

(ii) $T_{10}$, the tenth term

(iii) the value of $n$ such that $T_n = 200$

(iv) $S_{16}$, the sum to 16 terms.

**SOLUTION**

**5 (a) (i)**

$d = 8 - 2 = 6$

$d$ = Common difference = Any term − Previous term

**5 (a) (ii)**

General term: $T_n = a + (n-1)d$ ..... 2

$a = 2,\ d = 6,\ n = 10$

$T_n = a + (n-1)d$

$\Rightarrow T_{10} = (2) + (10-1)(6)$

$\Rightarrow T_{10} = (2) + (9)(6)$

$\Rightarrow T_{10} = 2 + 54 = 56$

**5 (a) (iii)**

Work out the formula for $T_n$ and then put it equal to 200 and solve for $n$.

$a = 2,\ d = 6,\ n = 10$

$T_n = a + (n-1)d$

$\Rightarrow T_n = 2 + (n-1)(6)$

$\Rightarrow T_n = 2 + 6n - 6$

$\Rightarrow T_n = 6n - 4$ → $T_n = 200 \Rightarrow 6n - 4 = 200$

$\Rightarrow 6n = 204$

$\Rightarrow n = \frac{204}{6} = 34$

**5 (a) (iv)**

Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ ..... 3

$a = 2,\ d = 6,\ n = 16$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{16} = \frac{16}{2}[2(2) + (16-1)(6)]$

$\Rightarrow S_{16} = 8[4 + (15)(6)]$

$\Rightarrow S_{16} = 8[4 + 90]$

$\Rightarrow S_{16} = 8[94] = 752$