SEQUENCES & SERIES (Q 5, PAPER 1)

2011

5. **(a)** The first term of a geometric sequence is 5 and the common ratio is 2.
Find the first four terms of the sequence.

(b) The first three terms of an arithmetic series are $7 + 4 + 1 + \ldots$.

(i) Find d, the common difference.

(ii) Find T_{15}, the fifteenth term of the series.

(iii) Find S_{15}, the sum of the first fifteen terms of the series.

(c) The first three terms of a geometric sequence are

$$h - 1,\ 2h \text{ and } 5h + 3,$$

where h is a real number greater than 1.

(i) Find the value of h.

(ii) The kth term of the sequence is 486. Find k.

SOLUTION

5 (a)

First term $a = 5$
Common ratio $r = 2$
Geometric sequence: 5, 10, 20, 40

5 (b) (i)

$7 + 4 + 1 + \ldots\ldots\ldots\ldots$

$a = 7$

$d = 4 - 7 = -3$

5 (b) (ii)

$a = 7$

$d = -3$

$n = 15$

$T_{15} = 7 + (15 - 1)(-3)$ General term: $\boxed{T_n = a + (n-1)d}$

$= 7 + (14)(-3)$

$= 7 - 42$

$= -35$

5 (b) (iii)

$a = 7$

$d = -3$

$n = 15$

$S_{15} = \frac{15}{2}[2(7)+(15-1)(-3)]$ Summing formula: $\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$

$= \frac{15}{2}[14+(14)(-3)]$

$= \frac{15}{2}[14-42]$

$= \frac{15}{2}[-28]$

$= 15[-14]$

$= -210$

5 (c) (i)

$h-1,\ 2h,\ 5h+3$ [Dividing consecutive terms in a geometric sequence gives you the same answer, the common ratio r.]

$$\frac{2h}{(h-1)} = \frac{(5h+3)}{2h}$$

$$\frac{2h \times 2h\,\cancel{(h-1)}}{\cancel{(h-1)}} = \frac{(5h+3)\times \cancel{2h}\,(h-1)}{\cancel{2h}}$$

$4h^2 = (5h+3)(h-1)$

$4h^2 = 5h(h-1)+3(h-1)$

$4h^2 = 5h^2-5h+3h-3$

$0 = 5h^2-4h^2-5h+3h-3$

$0 = h^2-2h-3$

$0 = (h-3)(h+1)$

$\therefore h = \cancel{-1},\ 3$ [You are told that $h > 0$, i.e. positive.]

5 (c) (ii)

Geometric sequence: 2, 6, 18 [Replace h by 3 to get the sequence.]
$a = 2,\ r = 3$

$T_k = 2\times 3^{k-1} \Rightarrow 486 = 2\times 3^{k-1}$ General term: $\boxed{T_n = ar^{n-1}}$

$243 = 3^{k-1}$

$3^5 = 3^{k-1}$

$\therefore 5 = k-1 \Rightarrow k = 6$