## Sequences \& Series (Q 5, Paper 1)

## 2010

5. (a) The first term of a geometric sequence is 4 and the common ratio is $0 \cdot 5$. Write down the first five terms of the sequence.
(b) In an arithmetic series, the first term is 6 and the fifth term is 22 .
(i) Find $d$, the common difference.
(ii) Find $T_{14}$, the fourteenth term.
(iii) Find $S_{20}$, the sum of the first twenty terms.
(c) In a geometric series, the fourth term is 9 and the seventh term is 243 .
(i) Find $r$, the common ratio.
(ii) Find $a$, the first term.
(iii) Find $S_{8}$, the sum of the first eight terms.

## Solution

5 (a)
$a=4, r=0.5$
Geometric Sequence: 4, 2, 1, 0.5, 0.25

To produce a geometric sequence, start with a number, $a$, and keep on multiplying by a number, $r$, forever.

5 (b) (i)
$a=6, T_{5}=22$
$T_{5}=a+4 d=22$
Ex. The fifty-sixth term of an arithmetic sequence: $T_{56}=a+55 d$
$\therefore(6)+4 d=22$
$4 d=16$
$\therefore d=4$
5 (b) (ii)
$T_{14}=a+13 d$
$\therefore T_{14}=6+13(4)=6+52=58$
5 (b) (iii)

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\(S_{20}=\frac{20}{2}[2(6)+(20-1) 4] \quad\) Summing formula: \(S_{n}=\frac{n}{2}[2 a+(n-1) d]\)
    \(=10[12+(19) 4]\)
    \(=10[12+76]\)
    \(=10[88]\)
    \(=880\)
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5 (c) (i)
$T_{4}=9 \Rightarrow a r^{3}=9 \ldots(\mathbf{1})$
$T_{7}=243 \Rightarrow a r^{6}=243 \ldots(\mathbf{2})$
Divide Eqn. (1) into Eqn. (2).

General term: $T_{n}=a r^{n-1}$

Ex. The tenth term of a geometric sequence: $T_{10}=a r^{9}$
$\frac{a r^{6}}{a r^{3}}=\frac{243}{9} \Rightarrow r^{3}=27$
$\therefore r=3$

## 5 (c) (ii)

Replace $r$ by its value in Eqn. (1) to find $a$.

$$
\begin{aligned}
& a r^{3}=9 \Rightarrow a(3)^{3}=9 \\
& 27 a=9 \\
& \therefore a=\frac{1}{3}
\end{aligned}
$$

## 5 (c) (iii)

$$
\begin{aligned}
S_{8} & =\frac{\frac{1}{3}\left(1-3^{8}\right)}{1-3}=\frac{\frac{1}{3}(1-6561)}{-2} \quad \text { Summing formula: } S_{n}=\frac{a\left(1-r^{n}\right)}{(1-r)} \\
& =\frac{\frac{1}{3}(-6560)}{-2}=\frac{1}{3}(3280)
\end{aligned}
$$

