SEQUENCES & SERIES (Q 5, PAPER 1)

2009

5 (a) The first term of a geometric sequence is 2 and the common ratio is 3. Find the second term of the sequence.
 (b) The first term of an arithmetic series is -2 and the second term is 4. (i) Find <i>d</i>, the common difference.
(ii) Find T_{10} , the tenth term of the series.
(iii) The <i>k</i> th term of the series is 292. Find <i>k</i> .
(iv) Find S_{20} , the sum of the first 20 terms of the series.
 (c) The first two terms of a geometric series are -6 + 12 + (i) Find <i>r</i>, the common ratio.
(ii) Find T_7 , the seventh term of the series.
(iii) Starting with the first term, how many terms of the series must be added to give a sum of 30?
SOLUTION 5 (a) To produce a GEOMETRIC SEQUENCE, start with a number, <i>a</i> , and have some kick in a number. Some some
r = 2, $r = 2$
a = 2, r = 5 Second term: $T_2 = ar = 2 \times 3 = 6$
5 (b) (i)
$a = -2, T_2 = 4$
Arithmetic sequence: $-2, 4, \dots$ d = 4 - (-2) = 4 + 2 = 6
5 (b) (ii) $T_{10} = -2 + (10 - 1)6$ = -2 + (9)6 = -2 + 54 = 52 $T_n = a + (n - 1)d$

5 (b) (iii) $T_k = 292 \Longrightarrow -2 + (k-1)6 = 292$ $T_n = a + (n-1)d$ -2+6k-6=2926k - 8 = 2926k = 300 $\therefore k = 50$ 5 (b) (iv) $S_{20} = \frac{20}{2} [2(-2) + (20 - 1)6] \qquad S_n = \frac{n}{2} [2a + (n - 1)d]$ =10[-4+19(6)]=10[-4+114]=10[110] = 11005 (c) (i) $r = \frac{12}{-6} = -2$ **COMMON RATIO** r = Any term ÷ Previous term 5 (c) (ii) $a = -6, \underline{r} = -2$ $T_n = ar^{n-1}$ $T_7 = -6 \times (-2)^{7-1}$ $= -6 \times (-2)^{6}$ $= -6 \times 64$ = -3845 (c) (iii) $S_n = 30 \Longrightarrow \frac{-6(1 - (-2)^n)}{1 - (-2)} = 30$ $S_n = \frac{a(1 - r^n)}{(1 - r)}$ $\frac{-6(1-(-2)^n)}{1+2} = 30$ $\frac{-6(1-(-2)^n)}{3} = 30$ $-2(1-(-2)^n)=30$ $(1-(-2)^n) = -15$ $1+15 = (-2)^n$ $16 = (-2)^n$ $\therefore n = 4$