SEQUENCES & SERIES (Q 5, PAPER 1)

2008

5 (a) Find the eleventh term of the arithmetic sequence 5, 14, 23 …

(b) The *n*th term of a geometric sequence is

$$T_n = \frac{3^n}{27}.$$

(i) Find a, the first term.

(ii) Find r, the common ratio.

(iii) The kth term of the sequence is 243. Find k.

(c) The sum of the first n terms of an arithmetic series is given by $S_n = n^2 - 16n$.

(i) Use S_1 and S_2 to find the first term and the common difference.

(ii) Find T_n, the nth term of the series.

(iii) Find the values of $n \in \mathbf{N}$ for which $S_n = -63$.

SOLUTION

5 (a)

$a = 5, d = 9, n = 11$ $\boxed{T_n = a + (n-1)d}$ 2

$T_{11} = 5 + (11-1)9$

$\Rightarrow T_{11} = 5 + 10 \times 9 = 5 + 90$

$\therefore T_{11} = 95$

5 (b) (i)

$$T_n = \frac{3^n}{27} \Rightarrow T_1 = a = \frac{3^1}{27} = \frac{1}{9}$$

5 (b) (ii)

$$T_n = \frac{3^n}{27} \Rightarrow T_2 = \frac{3^2}{27} = \frac{9}{27} = \frac{1}{3}$$

$$\therefore r = \frac{T_2}{T_1} = \frac{\frac{1}{3}}{\frac{1}{9}} = 3$$

5 (b) (iii)

$T_k = \frac{1}{9}(3)^{k-1} = 243$ $\boxed{T_n = ar^{n-1}}$ 4

$\Rightarrow \frac{1}{3^2} \times 3^{k-1} = 3^5$

$\Rightarrow 3^{k-3} = 3^5$

$\therefore k - 3 = 5 \Rightarrow k = 8$

POWER RULES

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$

5 (c) (i)

$S_n = n^2 - 16n$

$\Rightarrow S_1 = (1)^2 - 16(1) = 1 - 16 = -15$

$\Rightarrow S_2 = (2)^2 - 16(2) = 4 - 32 = -28$

$a = T_1 = S_1 = -15$ $S_1 = T_1$ for all sequences and series.

$T_2 = S_2 - S_1 = -28 - (-15) = -28 + 15 = -13$ $S_n - S_{n-1} = T_n$ 1

$d = T_2 - T_1 = -13 - (-15) = -13 + 15 = 2$ **COMMON DIFFERENCE** d = Any term – Previous term

5 (c) (ii)

$a = -15$

$d = 2$

$T_n = a + (n-1)d$ 2

$\Rightarrow T_n = -15 + (n-1)2$

$\Rightarrow T_n = -15 + 2n - 2$

$\therefore T_n = 2n - 17$

5 (c) (iii)

$S_n = -63$

$\Rightarrow n^2 - 16n = -63$

$\Rightarrow n^2 - 16n + 63 = 0$

$\Rightarrow (n-7)(n-9) = 0$

$\therefore n = 7, 9$