SEQUENCES & SERIES (Q 5, PAPER 1)

2007

5 (a) The nth term of a sequence is given by $T_n = 1 - n$.

(i) Find T_5, the fifth term.

(ii) Find $T_5 - T_{10}$ where T_{10} is the tenth term.

(b) The first term of an arithmetic series is 3 and the common difference is 4.

(i) Find, in terms of n, an expression for T_n, the nth term.

(ii) How many terms of the series are less than 200?

(iii) Find the sum of these terms.

(c) The first two terms of a geometric series are $\frac{1}{3} + \frac{1}{9} + \ldots$

(i) Find r, the common ratio.

(ii) Find an expression for S_n, the sum of the first n terms.

Write your answer in the form $\frac{1}{k}\left(1 - \frac{1}{3^n}\right)$ where $k \in \mathbf{N}$.

(iii) The sum of the first n terms of the geometric series $\frac{p}{3} + \frac{p}{9} + \ldots.$ is $1 - \frac{1}{3^n}$.

Find the value of p.

SOLUTION

5 (a) (i)

$T_n = 1 - n \Rightarrow T_5 = 1 - (5) = -4$

Put a bracket around n on each side and substitute in the little number (subscript).

5 (a) (ii)

$T_{10} = 1 - (10) = -9$

$\Rightarrow T_5 - T_{10} = -9 - (-4) = -9 + 4 = -5$

5 (b)

Arithmetic series

$a = 3$

$d = 4$

General term: $\boxed{T_n = a + (n-1)d}$ 2

5 (b) (i)

$T_n = a + (n-1)d = 3 + (n-1)4$

$= 3 + 4n - 4$

$= 4n - 1$

5 (b) (ii)

You are being asked to solve $T_n < 200$ for n.

$$T_n < 200 \Rightarrow 4n - 1 < 200$$
$$\Rightarrow 4n < 200 + 1$$
$$\Rightarrow 4n < 201$$
$$\Rightarrow n < \tfrac{201}{4}$$
$$\Rightarrow n < 50.25$$

n must be a whole positive number. Therefore, $n = 50$.

5 (b) (iii)

Summing formula: $\boxed{S_n = \tfrac{n}{2}[2a + (n-1)d]}$ 3

You are asked to find the sum of 50 terms ($n = 50$) where $a = 3$ and $d = 4$.

$$S_n = \tfrac{n}{2}[2a + (n-1)d]$$
$$\Rightarrow S_{50} = \tfrac{50}{2}[2(3) + (50-1)4]$$
$$\Rightarrow S_{50} = 25[6 + 49(4)]$$
$$\Rightarrow S_{50} = 25[6 + 196]$$
$$\Rightarrow S_{50} = 25[202]$$
$$\Rightarrow S_{50} = 5050$$

5 (c)

Geometric series: $\tfrac{1}{3} + \tfrac{1}{9} + ...$

5 (c) (i)

$$r = \frac{\tfrac{1}{9}}{\tfrac{1}{3}} = \tfrac{1}{9} \times \tfrac{3}{1} = \tfrac{1}{3}$$

$$\boxed{\text{Any term} \div \text{Previous term} = \frac{T_n}{T_{n-1}} = \text{Constant } (r)}$$

5 (c) (ii)

$a = \tfrac{1}{3}$

$r = \tfrac{1}{3}$

Summing formula: $\boxed{S_n = \dfrac{a(1-r^n)}{(1-r)}}$ 5

$$\therefore S_n = \frac{a(1-r^n)}{1-r} = \frac{(\tfrac{1}{3})(1-(\tfrac{1}{3})^n)}{1-(\tfrac{1}{3})}$$
$$= \frac{(\tfrac{1}{3})(1-(\tfrac{1}{3})^n)}{\tfrac{2}{3}}$$
$$= \frac{1}{2}\left(1 - \frac{1}{3^n}\right)$$

5 (c) (iii)

Geometric series: $\frac{p}{3}+\frac{p}{9}+\ldots$

$a=\frac{p}{3}$

$r=\dfrac{\frac{p}{9}}{\frac{p}{3}}=\frac{1}{3}$

$$S_n=\frac{a(1-r^n)}{1-r}=\frac{(\frac{p}{3})(1-(\frac{1}{3})^n)}{1-\frac{1}{3}}$$

$$=\frac{(\frac{p}{3})(1-(\frac{1}{3})^n)}{\frac{2}{3}}$$

$$=(\tfrac{p}{2})(1-(\tfrac{1}{3})^n)$$

$$=\frac{p}{2}\left(1-\frac{1}{3^n}\right)$$

You are told that the sum of the first n terms is given by $1-\dfrac{1}{3^n}$.

$$\therefore \frac{p}{2}\left(1-\frac{1}{3^n}\right)=1-\frac{1}{3^n}$$

$$\Rightarrow \frac{p}{2}=1$$

$$\Rightarrow p=2$$