SEQUENCES & SERIES (Q 5, PAPER 1)

2006

5 (a) The first term of an arithmetic sequence is 17 and the common difference is -8. Find, in terms of n, an expression for T_n, the nth term.

(b) The nth term of a geometric series is

$T_n = 4(\frac{1}{2})^n$.

(i) Find a, the first term.

(ii) Find r, the common ratio.

(iii) Write $4 - S_{10}$ in the form $\frac{1}{2^k}$, $k \in \mathbf{N}$, where S_{10} is the sum of the first ten terms.

(c) The first three terms of an arithmetic sequence are

$h+3,\ 5h-2,\ 6h-13$

where h is a real number.

(i) Find the value of h.

(ii) Hence, write down the value of each of the first three terms.

(iii) Find the value of the eleventh term.

SOLUTION

5 (a)

General term: $T_n = ar^{n-1}$ 4

The first term, a, is 17. The common difference is -8.

$a = 17$

$d = -8$

$T_n = a + (n-1)d$

$\Rightarrow T_n = 17 + (n-1)(-8)$

$\Rightarrow T_n = 17 - 8n + 8$

$\Rightarrow T_n = 25 - 8n$

5 (b) (i)

Replace n by 1 in the general term to find a.

$a = T_1$

$T_n = 4(\frac{1}{2})^n$

$\Rightarrow T_1 = 4(\frac{1}{2})^1 = 4(\frac{1}{2}) = 2$

5 (b) (ii)

To find the common ratio, r, find the second term, T_2, and then divide the second term by the first term.

$T_n = 4(\frac{1}{2})^n$

$\Rightarrow T_2 = 4(\frac{1}{2})^2 = 4(\frac{1}{4}) = 1$

$\therefore r = \dfrac{T_2}{T_1} = \frac{1}{2}$

5 (b) (iii)

$n = 10$

$a = 2$

$r = \frac{1}{2}$

Summing formula: $S_n = \dfrac{a(1-r^n)}{(1-r)}$ 5

$\therefore S_{10} = \dfrac{a(1-r^n)}{(1-r)}$

$\Rightarrow S_{10} = \dfrac{2(1-(\frac{1}{2})^{10})}{(1-\frac{1}{2})} = \dfrac{2(1-(\frac{1}{2})^{10})}{\frac{1}{2}}$

$\Rightarrow S_{10} = 4(1-(\frac{1}{2})^{10}) = 4 - 4(\frac{1}{2})^{10}$

$\therefore 4 - S_{10} = 4 - 4 + 4(\frac{1}{2})^{10}$

$= 4(\frac{1}{2})^{10} = 2^2 \times \dfrac{1}{2^{10}} = \dfrac{2^2}{2^{10}}$

$= \dfrac{1}{2^8}$

5 (c) $h+3,\ 5h-2,\ 6h-13$

5 (c) (i)

TEST: Any term – Previous term = $T_n - T_{n-1}$ = Constant (d)

As it is an arithmetic sequence, subtracting any two consecutive terms gives you the same constant. This constant is the common difference, d.

$5h-2-(h+3) = 6h-13-(5h-2)$

$\Rightarrow 5h-2-h-3 = 6h-13-5h+2$

$\Rightarrow 4h-5 = h-11$

$\Rightarrow 4h-h = -11+5$

$\Rightarrow 3h = -6$

$\Rightarrow h = -2$

5 (c) (ii)

$h+3,\ 5h-2,\ 6h-13$

$= (-2)+3,\ 5(-2)-2,\ 6(-2)-13$

$= -2+3,\ -10-2,\ -12-13$

$= 1,\ -12,\ -25$

5 (c) (ii)

General term: $T_n = ar^{n-1}$ **4**

$a = 1,$

$d = 12 - 1 = -13$

$n = 11$

$T_n = a + (n-1)d$

$\Rightarrow T_n = 1 + (11-1)(-13)$

$\Rightarrow T_n = 1 + (10)(-13)$

$\Rightarrow T_n = 1 - 130 = -129$