Sequences & Series (Q 5, Paper 1)

2005

5 (a) The first term of an arithmetic sequence is 9 and the second term is 13.

(i) Find the common difference.

(ii) Find the third term.

(b) The sum of the first n terms of an arithmetic series is given by

$S_n = n^2 + n.$

(i) Find a, the first term.

(ii) Find S_2, the sum of the first two terms.

(iii) Find d, the common difference.

(iv) Write down the first five terms of the series.

(c) In a geometric sequence of positive terms, the third term is $\frac{1}{4}$ and the fifth term is $\frac{1}{16}$.

(i) Find r, the common ratio.

(ii) Find a, the first term.

(iii) How many terms of the sequence are greater than 0·01?

Solution

5 (a)

Arithmetic sequence: 9, 13,...

5 (a) (i)

d = Common difference = Any term − Previous term

$d = 13 - 9 = 4$

5 (a) (ii)

Keep on adding the common difference, 4, to each term to get the next term.
Arithmetic sequence: 9, 13, 17, 21,.....
You can see the third term is 17.

5 (b)

$S_n = n^2 + n$

5 (b) (i)

$S_1 = T_1 = a$

$\Rightarrow S_1 = (1)^2 + (1) = 1 + 1 = 2$

$S_1 = T_1$ for all sequences and series.

5 (b) (ii)

$S_2 = (2)^2 + (2) = 4 + 2 = 6$

5 (b) (iii)

$S_n - S_{n-1} = T_n \Rightarrow S_2 - S_1 = T_2$

$\Rightarrow T_2 = 6 - 2 = 4$

$S_n - S_{n-1} = T_n$ 1

The first two terms of an arithmetic sequence are: 2, 4,...

d = Common difference = Any term − Previous term

$\therefore d = 4 - 2 = 2$

5 (b) (iv)

Keep on adding the common difference, 2, to each term to get the next term.
The first five terms of the arithmetic sequence are 2, 4, 6, 8, 10.

5 (c)

General term: $T_n = ar^{n-1}$ 4

Ex. The tenth term of a geometric sequence: $T_{10} = ar^9$

5 (c) (i)

$T_3 = ar^{n-1} = ar^2 = \frac{1}{4}$

$T_5 = ar^{n-1} = ar^4 = \frac{1}{16}$

Dividing $\Rightarrow \frac{ar^4}{ar^2} = \frac{\frac{1}{16}}{\frac{1}{4}} \Rightarrow r^2 = \frac{1}{16} \times \frac{4}{1} = \frac{1}{4} \Rightarrow r = \pm\frac{1}{2}$

As it is a geometric sequence of positive terms take $r = \frac{1}{2}$.

5 (c) (ii)

$ar^2 = \frac{1}{4} \Rightarrow a(\frac{1}{2})^2 = \frac{1}{4}$

$\Rightarrow a(\frac{1}{4}) = \frac{1}{4} \Rightarrow a = 1$

5 (c) (iii)

How many terms are greater that $0.01 = \frac{1}{100}$?

Write them out and count the number of the terms that are greater than 0.01.

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \ldots$

You can see that each of the first **seven** terms are greater than 0.01.