SEQUENCES & SERIES (Q 5, PAPER 1)

2004

5 (a) The first term of an arithmetic sequence is 40 and the common difference is -5.
Write down the first five terms of the sequence.

(b) The nth term of an arithmetic series is given by

$T_n = 1 + 5n$.

(i) The first term is a and the common difference is d.
Find the value of a and the value of d.

(ii) Find the value of n for which $T_n = 156$.

(iii) Find S_{12}, the sum of the first 12 terms.

(c) The first term of a geometric series is 1 and the common ratio is -4.

(i) Write down the first three terms of the series.

(ii) Find S_6, the sum of the first 6 terms.

(iii) Show that $16S_4 - 3 = S_6$, where S_4 is the sum of the first 4 terms.

SOLUTION

5 (a)

Start with 40 and keep on taking away 5.
Arithmetic sequence: 40, 35, 30, 25, 20,...

5 (b) (i)

Generate the first 2 terms of the arithmetic sequence by letting $n = 1$ and then letting $n = 2$.

$T_n = 1 + 5n$

$\Rightarrow T_1 = 1 + 5(1) = 1 + 5 = 6$

$\Rightarrow T_2 = 1 + 5(2) = 1 + 10 = 11$

Arithmetic sequence: 6, 11,....

d = Common difference = Any term − Previous term

First term $a = 6$

Common difference $d = 11 - 6 = 5$

5 (b) (ii)

$T_n = 156 \Rightarrow 1 + 5n = 156$

$\Rightarrow 5n = 156 - 1$

$\Rightarrow 5n = 155$

$\Rightarrow n = \frac{155}{5}$

$\Rightarrow n = 31$

5 (b) (iii)

Summing formula: $S_n = \frac{n}{2}[2a+(n-1)d]$ 3

$a = 6,\ d = 5,\ n = 12$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow S_{12} = \frac{12}{2}[2(6)+(12-1)(5)]$

$\Rightarrow S_{12} = 6[12+(11)(5)]$

$\Rightarrow S_{12} = 6[12+55]$

$\Rightarrow S_{12} = 6[67]$

$\Rightarrow S_{12} = 402$

5 (c) (i)

Write down the first term and keep on multiplying by the common ratio.

$a = 1,\ r = -4$

Geometric series: $1,\ -4,\ 16,..$

5 (c) (ii)

Summing formula: $S_n = \frac{a(1-r^n)}{(1-r)}$ 5

$n = 6,\ a = 1,\ r = -4$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$\Rightarrow S_6 = \frac{1(1-(-4)^6)}{(1-(-4))}$$

$$\Rightarrow S_6 = \frac{(1-4096)}{5}$$

$$\Rightarrow S_6 = \frac{-4095}{5} = -819$$

5 (c) (iii)

$n = 4,\ a = 1,\ r = -4$

$$16S_4 - 3 = 16\left(\frac{1(1-(-4)^4)}{(1-(-4))}\right) - 3$$

$$= 16\left(\frac{(1-256)}{5}\right) - 3$$

$$= 16\left(\frac{-255}{5}\right) - 3$$

$$= 16(-51) - 3$$

$$= -816 - 3 = -819$$