SEQUENCES & SERIES (Q 5, PAPER 1)

2003

5 (a) The first term of a geometric sequence is 4 and the commom ratio is 1·5.
Write down the next three terms of the sequence.

(b) The first two terms of a geometric series are 32 + 8 +...
(i) What us the value of r, the common ratio?

(ii) Find an expression for S_n, the sum of the first n terms.

(iii) Find S_{10}, the sum of the first 10 terms.
Given your answer correct to four decimal places.

(c) The fifth term of an arithmetic series is 21 and the tenth term is 11.
(i) Find the first term and the common difference.

(ii) Find the sum of the first twenty terms.

(iii) For what value of $n > 0$ is the sum of the first n terms equal to zero?

SOLUTION

5 (a)
Write down the first term and keep on multiplying by the common ratio, r, to generate the terms of the geometric sequence.

4, 6, $\frac{27}{2}$,...

5 (b) (i)
Geometric series: 32 + 8 + ...

r = Common ratio = Any term ÷ Previous term

$\therefore r = \frac{8}{32} = \frac{1}{4}$

5 (b) (ii)

$a = 32,\ r = \frac{1}{4}$

Summing formula: $S_n = \frac{a(1-r^n)}{(1-r)}$ 5

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$\Rightarrow S_n = \frac{32(1-(\frac{1}{4})^n)}{(1-\frac{1}{4})}$$

$$\Rightarrow S_n = \frac{32(1-(\frac{1}{4})^n)}{\frac{3}{4}}$$

$$\Rightarrow S_n = \frac{128}{3}(1-(\frac{1}{4})^n)$$

5 (b) (iii)

$S_{10} = \frac{128}{3}(1-(\frac{1}{4})^{10}) = 42.6666$ [Use calculator]

5 (c) (i)

General term: $T_n = a + (n-1)d$ 2

Ex. The fifty-sixth term of an arithmetic sequence: $T_{56} = a + 55d$

$T_5 = a + 4d = 21....(\mathbf{1})$

$T_{10} = a + 9d = 11....(\mathbf{2})$ ← Solve simultaneously by subtracting.

$-5d = 10 \Rightarrow d = -2$

Substitute this value of d back into Eqn. (**1**): $a + 4(-2) = 21 \Rightarrow a - 8 = 21 \Rightarrow a = 29$

5 (c) (ii)

Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ 3

$a = 29,\ d = -2,\ n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_n = \frac{10}{2}[2(29) + (10-1)(-2)]$

$\Rightarrow S_n = 5[58 + (9)(-2)]$

$\Rightarrow S_n = 5[58 - 18]$

$\Rightarrow S_n = 5[40] = 200$

5 (c) (iii)

Put $S_n = 0$ and solve for n.

$a = 29,\ d = -2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_n = \frac{n}{2}[2(29) + (n-1)(-2)] = 0$

$\Rightarrow \frac{n}{2}[58 + (n-1)(-2)] = 0$

$\Rightarrow \frac{n}{2}[58 - 2n + 2] = 0$

$\Rightarrow \frac{n}{2}[60 - 2n] = 0$

$\Rightarrow n[30 - n] = 0$ [Set each factor equal to zero and solve for n.]

$\Rightarrow n = 0,\ 30$

As $n > 0$, the answer is $n = 30$.