SEQUENCES & SERIES (Q 5, PAPER 1)

2002

5 (a) Write down the next three terms in each of the following arithmetic sequences

(i) $-10,\ -8,\ -6, \ldots\ldots$

(ii) $4.1,\ 4.7,\ 5.3, \ldots\ldots$

(b) The sum of the first n terms of an arithmetic series is given by

$S_n = \frac{3n}{2}(n+3)$.

(i) Calculate the first term of the series.

(ii) By calculating S_9 and S_{10}, find T_{10} (the tenth term of the series).

(c) The first three terms of a geometric sequence are

$k-3,\ 2k-4,\ 4k-3, \ldots\ldots$

where k is a real number.

(i) Find the value of k.

(ii) Hence, write down the value of each of the first four terms of the sequence.

SOLUTION

5 (a) (i)

d = Common difference = Any term − Previous term

$d = -8-(-10) = -8+10 = +2$

Keep on adding on 2 to each term: $-10,\ -8,\ -6,\ -4,\ -2,\ 0$

5 (a) (ii)

$d = 4.7-4.1 = 0.6$

Keep on adding 0.6 on to each term: $4.1,\ 4.7,\ 5.3,\ 5.9,\ 6.5,\ 7.1$

5 (b) (i)

$S_1 = T_1$ for all sequences and series.

$S_n = \frac{3n}{2}(n+3)$

$\Rightarrow S_1 = \frac{3(1)}{2}((1)+3)$

$\Rightarrow S_1 = \frac{3}{2}(4) = 6$

5 (b) (ii)

$S_n - S_{n-1} = T_n$ 1

$S_n = \frac{3n}{2}(n+3)$

$\therefore S_{10} = \frac{3(10)}{2}((10)+3)$

$\Rightarrow S_{10} = \frac{30}{2}(13) = 195$

$\therefore S_9 = \frac{3(9)}{2}((9)+3)$

$\therefore S_9 = \frac{27}{2}(12) = 162$

$S_{10} - S_9 = T_{10}$

$\Rightarrow 195 - 162 = T_{10}$

$\therefore T_{10} = 33$

5 (c) (i)

As it is a geometric sequence, dividing any two consecutive terms gives you the same constant. This constant is the common ratio, r.

$\frac{(2k-4)}{(k-3)} = \frac{(4k-3)}{(2k-4)}$ [Multiply both sides by $(k-3)(2k-4)$ or cross multiply.]

$\Rightarrow (2k-4)(2k-4) = (4k-3)(k-3)$ [Multiply out the brackets.]

$\Rightarrow 4k^2 - 8k - 8k + 16 = 4k^2 - 12k - 3k + 9$

$\Rightarrow -16k + 16 = -15k + 9$

$\Rightarrow 16 - 9 = -15k + 16k$

$\Rightarrow 7 = k$

5 (c) (ii)

$k-3,\ 2k-4,\ 4k-3, \ldots$ [Replace k by its value.]

$= (7)-3,\ 2(7)-4,\ 4(7)-3, \ldots$

$= 7-3,\ 14-4,\ 28-3, \ldots$

$= 4,\ 10,\ 25, \ldots$

To get the fourth term, find the common ratio.

$\therefore r = \frac{10}{4} = \frac{5}{2}$

Multiply the third term by r.

$\therefore T_4 = \frac{5}{2} \times 25 = \frac{125}{2}$

The first four terms are: $4,\ 10,\ 25,\ \frac{125}{2}$