## Sequences \& Series (Q 5, Paper 1)

2001
5 (a) $5,13,21,29, \ldots$. is an arithmetic sequence.
Which term of the sequence is 813 ?
(b) The $n$th term of a geometric series is given by $T_{n}=3^{n}$.
(i) What is the value of $a$, the first term?
(ii) What is the value of $r$, the common ratio?
(iii) Show that $S_{10}$, the sum of the first ten terms, is $\frac{3}{2}\left(3^{10}-1\right)$.
(c) The sum of the first n terms of an arithmetic series is given by

$$
S_{n}=4 n^{2}-8 n
$$

(i) Use $S_{1}$ and $S_{2}$ to find the first term and the common difference.
(ii) Starting with the first term, how many terms of the series must be added to give a sum of 252 ?

## Solution

5 (a)

$$
\text { General term: } T_{n}=a+(n-1) d \ldots \ldots .2
$$

$a=5, d=8$
$T_{n}=a+(n-1) d$
$\Rightarrow T_{n}=5+(n-1)(8)$
$\Rightarrow T_{n}=5+8 n-8$
$\Rightarrow T_{n}=8 n-3$
Put the general term, $T_{n}$, equal to 813 and solve for $n$.
$\therefore 8 n-3=813$
$\Rightarrow 8 n=816$
$\Rightarrow n=102$
5 (b) (i)
$T_{n}=3^{n}$
$\therefore T_{1}=a=3^{1}=3$
5 (b) (ii)

$$
r=\text { Common ratio }=\text { Any term } \div \text { Previous term }
$$

Find the second term, $T_{2}$, and then find the common ratio $r$ by dividing the second term by the first term.
$T_{2}=3^{2}=9$
$\therefore r=\frac{9}{3}=3$

## 5 (b) (iii)

$a=3, r=3, n=10$ Summing formula: $S_{n}=\frac{a\left(1-r^{n}\right)}{(1-r)}$
$S_{n}=\frac{a\left(1-r^{n}\right)}{(1-r)}$
$\Rightarrow S_{10}=\frac{3\left(1-3^{10}\right)}{(1-3)}$
$\Rightarrow S_{10}=\frac{3\left(1-3^{10}\right)}{-2}$
$\Rightarrow S_{10}=-\frac{3}{2}\left(1-3^{10}\right)$
$\Rightarrow S_{10}=\frac{3}{2}\left(3^{10}-1\right)$
5 (c) (i)
$S_{n}=4 n^{2}-8 n$
$\therefore S_{1}=4(1)^{2}-8(1)=4 \times 1-8=4-8=-4$
$\therefore S_{2}=4(2)^{2}-8(2)=4 \times 4-16=16-16=0$
$S_{1}=a=-4$
$S_{2}-S_{1}=T_{2}=0-(-4)=0+4=4$
Arithmetic sequence: $-4,4, \ldots$
$d=4-(-4)=4+4=8$
5 (c) (ii)
Put $S_{n}$ equal to 252 and solve for $n$.
$S_{n}=252$
$\Rightarrow 4 n^{2}-8 n=252$
$\Rightarrow 4 n^{2}-8 n-252=0$
$\Rightarrow n^{2}-2 n-63=0$
$\Rightarrow(n-9)(n+7)=0$
$\therefore n=9,-7$
Ignore the negative solution as $n$ must be positive.
Therefore, 9 terms must be added together to give a total of 252.

