SEQUENCES & SERIES (Q 5, PAPER 1)

2000

5 (a) The *n*th term of a sequence is given by $T_n = n^2 + 1$.

(i) Write down the first three terms of the sequence.

(ii) Show that $T_1 + T_2 + T_3 = T_4$.

(b) The first term of a geometric series is 1 and the common ratio is $\frac{11}{10}$.

(i) Write down the second, thirds and fourth terms of the series.

(ii) Calculate S_4, the sum of the first four terms. Give your answer as a decimal.

(c) The first three terms of an arithmetic series are 5 + 10 + 15 + ...

(i) Find, in terms of *n*, an expression for T_n, the *n*th term.

(ii) Find, in terms of *n*, an expression for S_n, the sum to *n* terms.

(iii) Using your expression for S_n, find the sum of the natural numbers that are both multiples of 5 and smaller than 1000.

SOLUTION

5 (a) (i)

$T_n = n^2 + 1$

$\Rightarrow T_1 = (1)^2 + 1 = 1 + 1 = 2$

$\Rightarrow T_2 = (2)^2 + 1 = 4 + 1 = 5$

$\Rightarrow T_3 = (3)^2 + 1 = 9 + 1 = 10$

Arithmetic sequence: 2, 5, 10

5 (a) (ii)

$T_n = n^2 + 1$

$\Rightarrow T_4 = (4)^2 + 1 = 16 + 1 = 17$

$T_1 + T_2 + T_3 = 2 + 5 + 10 = 17$

$\therefore T_1 + T_2 + T_3 = T_4$

5 (b) (i)

To generate the terms of a geometric series multiply each term by the ratio to get the next term.

$a = T_1 = 1$

$T_2 = 1 \times \frac{11}{10} = \frac{11}{10}$

$T_3 = \frac{11}{10} \times \frac{11}{10} = \frac{121}{100}$

$T_4 = \frac{121}{100} \times \frac{11}{10} = \frac{1331}{1000}$

5 (b) (ii)

$S_4 = T_1 + T_2 + T_3 + T_4 = 1 + \frac{11}{10} + \frac{121}{100} + \frac{1331}{1000} = 1 + 1.1 + 1.21 + 1.331 = 1.641$

5 (c) (i)

$a = 5,\ d = 5$ General term: $T_n = a + (n-1)d$ 2

$T_n = a + (n-1)d$

$\Rightarrow T_n = 5 + (n-1)(5)$

$\Rightarrow T_n = 5 + 5n - 5$

$\Rightarrow T_n = 5n$

5 (c) (ii)

$a = 5,\ d = 5$ Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ 3

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_n = \frac{n}{2}[2(5) + (n-1)(5)]$

$\Rightarrow S_n = \frac{n}{2}[10 + 5n - 5]$

$\Rightarrow S_n = \frac{n}{2}[5n + 5]$

5 (c) (iii)

The series contains terms that are multiples of 5. Put T_n equal to 1000 and solve for n. This will tell you the number of terms that are smaller than 1000. Now you know the number of terms you need to add together.

$T_n = 5n$

$\Rightarrow 5n = 1000$

$\Rightarrow n = 200$

The 200th. term is 1000. Therefore, 199 terms are less than 1000. Add together the first 199 terms.

$a = 5,\ d = 5,\ n = 199$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{199} = \frac{199}{2}[2(5) + (199-1)(5)]$

$\Rightarrow S_{199} = \frac{199}{2}[10 + (198)(5)]$

$\Rightarrow S_{199} = \frac{199}{2}[10 + 990]$

$\Rightarrow S_{199} = \frac{199}{2}[1000]$

$\Rightarrow S_{199} = 199[500] = 99,500$