SEQUENCES & SERIES (Q 5, PAPER 1)

1999

5 (a) The *n*th term of a sequence is given by

$$T_n = \frac{n}{n+1}.$$

(i) Find T_2, the second term.

(ii) Show that $T_2 + T_3 > 1$.

(b) The first two terms of a geometric series are $2 + \frac{2}{3} + \ldots$

(i) Find *r*, the common ratio.

(ii) Write down the third and fourth terms of the series.

(iii) Show that S_6, the sum to 6 terms, is $3 - \frac{1}{3^5}$.

(c) The *n*th term of a series is given by

$T_n = 4n + 1$.

(i) Write down, in terms of *n*, an expression for T_{n-1}, the $(n-1)$st. term.

(ii) Show that the series is arithmetic.

(iii) Find S_{20}, the sum of the first 20 terms of the series.

SOLUTION

5 (a) (i)

$$T_n = \frac{n}{n+1}$$

$$\Rightarrow T_2 = \frac{(2)}{(2)+1} = \frac{2}{3}$$

5 (a) (ii)

$$T_n = \frac{n}{n+1}$$

$$\Rightarrow T_3 = \frac{(3)}{(3)+1} = \frac{3}{4}$$

$$\therefore T_2 + T_3 = \tfrac{2}{3} + \tfrac{3}{4} = \tfrac{17}{12} = 1\tfrac{5}{12} > 1$$

5 (b) (i)

r = Common ratio = Any term ÷ Previous term

$$r = \frac{\frac{2}{3}}{2} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

5 (b) (ii)

To generate the terms of a geometric sequence, keep on multiplying each term by the common ratio r to get the next term.

$$T_3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

$$T_4 = \frac{2}{9} \times \frac{1}{3} = \frac{2}{27}$$

5 (b) (iii)

$a = 2,\ r = \frac{1}{3},\ n = 6$ Summing formula: $S_n = \frac{a(1-r^n)}{(1-r)}$ 5

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$\Rightarrow S_6 = \frac{2(1-(\frac{1}{3})^6)}{(1-(\frac{1}{3}))}$$

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$$\Rightarrow S_6 = \frac{2(1-(\frac{1}{3})^6)}{\frac{2}{3}} \quad [\textbf{Note: } \frac{2}{\frac{2}{3}} = 2 \times \frac{3}{2} = 3\,]$$

$$\Rightarrow S_6 = 3(1-(\tfrac{1}{3})^6)$$

$$\Rightarrow S_6 = 3 - 3(\tfrac{1}{3})^6 \quad [\textbf{Note: } 3(\tfrac{1}{3})^6 = 3^1 \times \frac{1}{3^6} = \frac{1}{3^5}\,]$$

$$\Rightarrow S_6 = 3 - \frac{1}{3^5}$$

5 (c) (i)

Replace n by $(n-1)$.

$$T_n = 4n + 1$$

$$\Rightarrow T_{n-1} = 4(n-1) + 1$$

$$\Rightarrow T_{n-1} = 4n - 4 + 1$$

$$\Rightarrow T_{n-1} = 4n - 3$$

5 (c) (ii)

TEST THAT A SERIES IS ARITHMETIC: Any term – Previous term = $T_n - T_{n-1}$ = Constant (d)

$$T_n - T_{n-1} = 4n + 1 - (4n - 3)$$

$$= 4n + 1 - 4n + 3$$

$$= 4$$

Therefore, the series is arithmetic because 4 is a constant. This constant is the common difference d.

5 (c) (iii)

Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ 3

You need to find the first term, a. You do this by letting $n = 1$ in the general term.

$T_n = 4n + 1$

$\Rightarrow T_1 = 4(1) + 1 = 5$

$a = 5,\ d = 4,\ n = 20$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{20} = \frac{20}{2}[2(5) + (20-1)(4)]$

$\Rightarrow S_{20} = 10[10 + (19)(4)]$

$\Rightarrow S_{20} = 10[10 + 76]$

$\Rightarrow S_{20} = 10[86] = 860$