SEQUENCES & SERIES (Q 5, PAPER 1)

1998

5 (a) The first two terms of an arithmetic sequence are 17, 13,...
Find
(i) d, the common difference

(ii) T_7, the seventh term.

(b) The nth term of a geometric sequence is

$$T_n = \frac{2^n}{3^n}.$$

(i) Find the first three terms of the sequence.

(ii) Show that S_5, the sum of the first five terms, is $\frac{422}{243}$.

(c) The first three terms of an arithmetic series are
$2d + 3d + 4d + ...$
where d is a real number.
(i) Find, in terms of d, an expression for T_{10}, the tenth term.

(ii) Find, in terms of d, an expression for S_{10}, the sum to 10 terms.

(iii) If $S_{10} - T_{10} = 162$, find the value of d and write down the first four terms of the series.

SOLUTION

5 (a) (i)

d = Common difference = Any term − Previous term

$d = 13 - 17 = -4$

5 (a) (ii)

Keep on adding -4 to each term to generate the next term. Keep going till you get to the seventh term.

Arithmetic sequence: $17, 13, 9, 5, 1, -3, -7, ...$

The seventh term $T_7 = -7$.

5 (b) (i)

$$T_n = \frac{2^n}{3^n}$$

$$\therefore T_1 = \frac{2^{(1)}}{3^{(1)}} = \frac{2}{3}$$

$$\therefore T_2 = \frac{2^{(2)}}{3^{(2)}} = \frac{4}{9}$$

$$\therefore T_3 = \frac{2^{(3)}}{3^{(3)}} = \frac{8}{27}$$

Geometric sequence: $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, ...$

5 (b) (ii)

Summing formula: $S_n = \frac{a(1-r^n)}{(1-r)}$ 5

r = Common ratio = Any term ÷ Previous term

$r = \frac{\frac{4}{9}}{\frac{2}{3}} = \frac{4}{9} \times \frac{3}{2} = \frac{2}{3}$

$a = \frac{2}{3},\ r = \frac{2}{3},\ n = 5$

$S_n = \frac{a(1-r^n)}{(1-r)}$

$\Rightarrow S_5 = \frac{(\frac{2}{3})(1-(\frac{2}{3})^5)}{(1-(\frac{2}{3}))}$

$\Rightarrow S_5 = \frac{(\frac{2}{3})(1-\frac{32}{243})}{\frac{1}{3}}$

$\Rightarrow S_5 = 2(\frac{211}{243}) = \frac{422}{243}$

5 (c) (i)

General term: $T_n = a + (n-1)d$ 2

Arithmetic sequence: $2d$, $3d$, $4d$,...

d = Common difference = Any term − Previous term

Common difference $3d - 2d = d$

$a = 2d,\ d = d,\ n = 10$

$T_n = a + (n-1)d$

$\Rightarrow T_{10} = 2d + (10-1)d$

$\Rightarrow T_{10} = 2d + 9d = 11d$

5 (c) (ii)

Summing formula: $S_n = \frac{n}{2}[2a + (n-1)d]$ 3

$a = 2d,\ d = d,\ n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2(2d) + (10-1)d]$

$\Rightarrow S_{10} = 5[4d + 9d]$

$\Rightarrow S_{10} = 5[13d] = 65d$

5 (c) (iii)

$S_{10} - T_{10} = 162$

$\Rightarrow 65d - 11d = 162$

$\Rightarrow 54d = 162$

$\Rightarrow d = \frac{162}{54} = 3$

Replace d by 3 to generate the series:
Arithmetic series: 6 + 9 + 12 + 15