SEQUENCES & SERIES (Q 5, PAPER 1)

1997

5 (a) $T_1 + T_2 + T_3 + ...$ is a geometric series.
The first term, T_1, is 1 and the common ratio is 2.
Show that
$$T_3 + T_5 = 2(T_2 + T_4).$$

(b) The first four terms of an arithmetic sequence are given as
$$a, -4, b, 6, ...$$
Find
(i) the value of a and the value of b

(ii) T_5, the fifth term.

(c) In an arithmetic series
$$S_n = n^2 + n,$$
where S_n is the sum to the first n terms.
Write down
(i) S_{10}, the sum to 10 terms

(ii) S_{11}, the sum to 11 terms

(iii) T_{11}, the 11th. term.

SOLUTION

5 (a)

> To produce a GEOMETRIC SEQUENCE, start with a number, a, and keep on multiplying by a number, r, forever.

Geometric series: 1 + 2 + 4 + 8 + 16 + ...
$T_1 = 1$, $T_2 = 2$, $T_3 = 4$, $T_4 = 8$, $T_5 = 16$

$T_3 + T_5 = 4 + 16 = 20$

$2(T_2 + T_4) = 2(2 + 8) = 2(10) = 20$

$\therefore T_3 + T_5 = 2(T_2 + T_4)$

5 (b) (i)

> d = Common difference = Any term − Previous term

The difference between the fourth and second terms is $2d$.

$2d = 6 - (-4) = 6 + 4 = 10$

$\therefore d = 5$

You keep on adding on 5 to generate each term in the sequence.
Arithmetic sequence: $-9, -4, 1, 6, ...$

$\therefore a = -9, b = 1$

5 (b) (ii)

$T_5 = 11$ [Add 5 on to the fourth term to get the fifth term.]

5 (c) (i)

$S_n = n^2 + n$

$\Rightarrow S_{10} = (10)^2 + (10)$

$\Rightarrow S_{10} = 100 + 10$

$\Rightarrow S_{10} = 110$

5 (c) (ii)

$S_n = n^2 + n$

$\Rightarrow S_{11} = (11)^2 + (11)$

$\Rightarrow S_{11} = 121 + 11$

$\Rightarrow S_{11} = 132$

5 (c) (iii)

$\boxed{S_n - S_{n-1} = T_n}$ 1

$T_{11} = S_{11} - S_{10}$

$\Rightarrow T_{11} = 132 - 110 = 22$