# SEQUENCES & SERIES (Q 5, PAPER 1)

## 1996

5 (a) The first two terms of an arithmetic series are given as
2 + 8 +.....
Find
(i) $d$, the common difference

(ii) $T_{10}$, the tenth term

(iii) the value of $n$ such that $T_n = 200$

(iv) $S_{16}$, the sum to 16 terms.

(b) The $n$th term, $T_n$, of a geometric series is

$$T_n = 3^{n-1}.$$

Find
(i) $T_1$, the first term

(ii) $r$, the common ratio

(iii) $S_n$, the sum to $n$ terms.
Investigate if

$$2S_n - T_n = 2T_n - 1.$$

**SOLUTION**

**5 (a) (i)**

$d = 8 - 2 = 6$

$d$ = Common difference = Any term − Previous term

**5 (a) (ii)**

General term: $T_n = a + (n-1)d$ ..... 2

$a = 2,\ d = 6,\ n = 10$

$T_n = a + (n-1)d$

$\Rightarrow T_{10} = (2) + (10-1)(6)$

$\Rightarrow T_{10} = (2) + (9)(6)$

$\Rightarrow T_{10} = 2 + 54 = 56$

**5 (a) (iii)**

Work out the formula for $T_n$ and then put it equal to 200 and solve for $n$.

$a = 2,\ d = 6,\ n = 10$

$T_n = a + (n-1)d$

$\Rightarrow T_n = 2 + (n-1)(6)$

$\Rightarrow T_n = 2 + 6n - 6$

$\Rightarrow T_n = 6n - 4$ → $T_n = 200 \Rightarrow 6n - 4 = 200$

$\Rightarrow 6n = 204$

$\Rightarrow n = \frac{204}{6} = 34$

**5 (a) (iv)**

Summing formula: $\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$ ..... 3

$a = 2,\ d = 6,\ n = 16$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow S_{16} = \frac{16}{2}[2(2)+(16-1)(6)]$

$\Rightarrow S_{16} = 8[4+(15)(6)]$

$\Rightarrow S_{16} = 8[4+90]$

$\Rightarrow S_{16} = 8[94] = 752$

**5 (b) (i)**

$T_n = 3^{n-1} \Rightarrow T_1 = 3^{(1)-1} = 3^0 = 1$

**5 (b) (ii)**

$\boxed{r = \text{Common ratio} = \text{Any term} \div \text{Previous term}}$

Find the second term $T_2$ and then divide the second term by the first term to find $r$.

$T_n = 3^{n-1} \Rightarrow T_2 = 3^{(2)-1} = 3^1 = 3$

$\Rightarrow r = \dfrac{T_2}{T_1} = \dfrac{3}{1} = 3$

**5 (b) (iii)**

Summing formula: $\boxed{S_n = \dfrac{a(1-r^n)}{(1-r)}}$ ..... 5

$a = 1,\ r = 3$

$S_n = \dfrac{a(1-r^n)}{(1-r)}$

$\Rightarrow S_n = \dfrac{1(1-3^n)}{(1-3)}$

$\Rightarrow S_n = \dfrac{1(1-3^n)}{-2}$

$\Rightarrow S_n = \frac{1}{2}(3^n - 1)$

| *LHS* | *RHS* |
|---|---|
| $2S_n - T_n$ | $2T_n - 1$ |
| $= 2[\frac{1}{2}(3^n-1)] - 3^{n-1}$ | $= 2(3^{n-1}) - 1$ |
| $= 3^n - 1 - 3^{n-1}$ | |
| $= (3^n - 3^{n-1}) - 1$ | |
| $= 3^{n-1}(3^1 - 1) - 1$ | |
| $= 3^{n-1}(2) - 1$ | |

$\therefore 2S_n - T_n = 2T_n - 1$