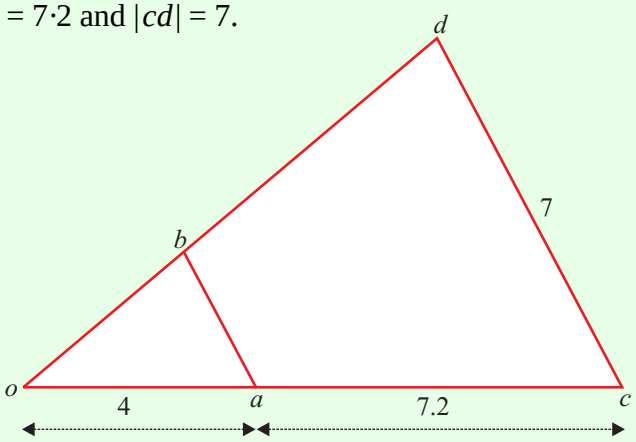
GEOMETRY (Q 4, PAPER 2)

LESSON NO. 5: ENLARGEMENTS

2007

4 (c) The triangle *ocd* is the image of the triangle *oab* under an enlargement with centre *o*. $|oa| = 4$, $|ac| = 7{\cdot}2$ and $|cd| = 7$.



(i) Find the scale factor of the enlargement.

(ii) Find $|ab|$.

(iii) The area of the triangle *oab* is 4·5 square units.
Find the area of the triangle *ocd*.

SOLUTION

4 (c) (i)

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \ldots\ldots \text{(1)}$$

$$k = \frac{|oc|}{|oa|} = \frac{4+7.2}{4} = \frac{11.2}{4} = 2.8$$

4 (c) (ii)

$$k = \frac{|cd|}{|ab|} \Rightarrow 2.8 = \frac{7}{|ab|}$$

$$\therefore |ab| = \frac{7}{2.8} = 2.5$$

4 (c) (iii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{(2)}$$

Object area (triangle *oab*) = 4.5 square units
Image area (triangle *ocd*) = ?

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \Rightarrow 2.8^2 = \frac{|\text{Image area}|}{|4.5|}$$

$$\therefore |\text{Image area}| = 2.8^2 \times 4.5 = 35.28 \text{ square units}$$

2006

4 (c) (i) Construct a triangle abc in which $|ab| = 6.5$ cm, $|bc| = 2.5$ cm and $|ac| = 6$ cm.

(ii) Construct the image of the triangle abc under the enlargement of scale factor 1.8 and centre c.

(iii) Given that the area of triangle abc is 7.5 cm^2, find the area of the image triangle.

SOLUTION

4 (c) (i)

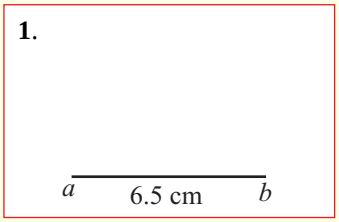
Draw a triangle abc whose sides have lengths of $|ab| = 6.5$ cm, $|ac| = 6$ cm and $|bc| = 2.5$ cm.

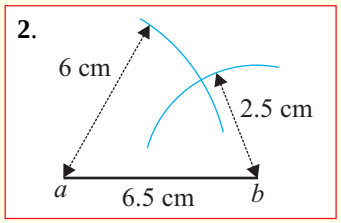
STEP 1: Draw a base using the longest side. Use a ruler to measure out a line segment $[ab]$ of length 6.5 cm.

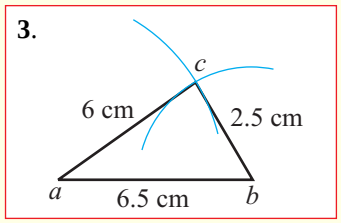
STEP 2. Take a compass and use a ruler to measure out a length of 6 cm. Put the point of the compass at a and draw out an arc of the circle. Do the same for the third side. Using your compass again measure out a length of 2.5 cm. Put the point of the compass at b and draw out an arc of the circle so that it intersects with the other arc.

STEP 3. c is the point of intersection of the two arcs. Join a to c and b to c to complete the construction.

NOTE: Only approximate lengths are shown in the diagrams.





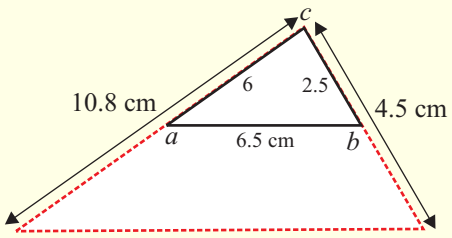


4 (c) (ii)

Multiply the lengths of the lines by the scale factor of 1.8 to find the lengths of their images.

Length of image of $bc = 2.5 \times 1.8 = 4.5$ cm

Length of image of $ac = 6 \times 1.8 = 10.8$ cm



4 (c) (iii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{(2)}$$

Object area (triangle abc) = 7.5 cm^2

Image area (big triangle) = ?

Scale factor $k = 1.8$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \Rightarrow 1.8^2 = \frac{|\text{Image area}|}{|7.5|}$$

$$\therefore |\text{Image area}| = 1.8^2 \times 7.5 = 24.3 \text{ cm}^2$$

2005

4 (c) (i) Draw a square *opqr* with sides 8 cm.

(ii) Draw the image of this square under the enlargement with centre *o* and scale factor 0.25.

(iii) Calculate the area of this image square.

(iv) Under another enlargement the area of the image of the square *opqr* is 100 cm^2. What is the scale factor of this enlargement?

SOLUTION

4 (c) (i)

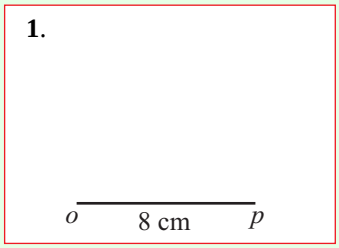
Construct a square *opqr* with side of length 8 cm.

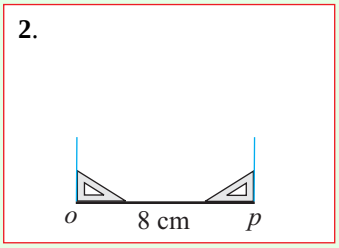
STEP 1. Using a ruler draw a side *op* of length 8 cm.

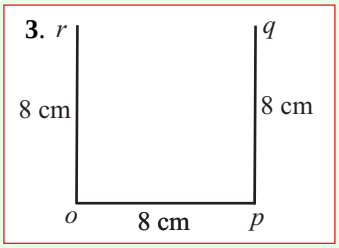
STEP 2. Place the right-angle of a set square on point *o* and draw a light line. Do the same at point *p*.

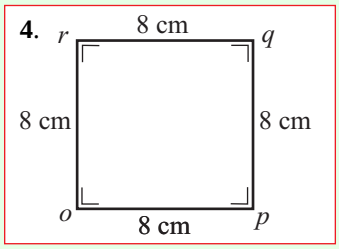
STEP 3. Using the ruler draw a line, *or*, of length 8 cm from point *o* through the light line. Do the same at point *p* drawing line *pq*.

STEP 4. Complete the square by joining *r* to *q* to form line *rq*. Using your set square, make sure all the angles are right-angled. Using your ruler, make sure each side is of length 8 cm.







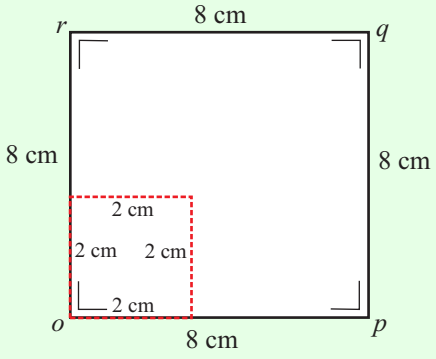


4 (c) (ii)

Scale factor $k = 0.25$

The lengths of the image of each side $8 \times 0.25 = 2$ cm

NOTE: All lengths shown are approximate. When you are doing the question the lengths must be the exact measure.



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4 (c) (iii)

Area of the image square $= 2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$

4 (c) (iv)

Scale factor k = ?

$|\text{Object Area}| = 8 \text{ cm} \times 8 \text{ cm} = 64 \text{ cm}^2$

$|\text{Image Area}| = 100 \text{ cm}^2$

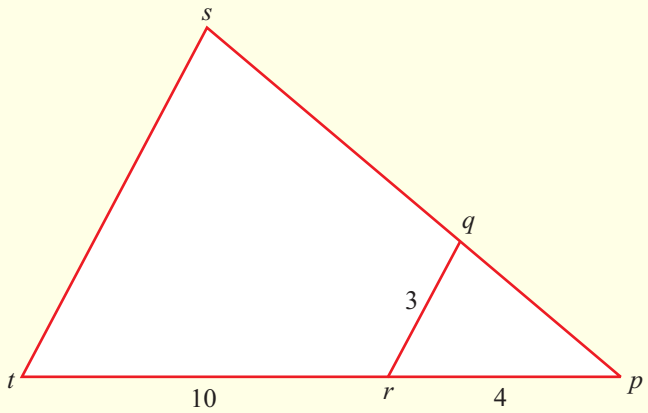
$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|}$$ 2

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} = \frac{100}{64} = \frac{25}{16}$$

$$\therefore k = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1.25$$

2004

4 (c) The triangle *pst* is the image of the triangle *pqr* under an enlargement with centre *p*.



$|pr| = 4$, $|rt| = 10$ and $|qr| = 3$.

(i) Find the scale factor of the enlargement.

(ii) Find $|st|$.

(iii) The area of the triangle *pqr* is 5 square units.
Find the area of the quadrilateral *qstr*.

SOLUTION

4 (c) (i)

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \ldots\ldots \text{ (1)}$$

$$k = \frac{|pt|}{|pr|} = \frac{4+10}{4} = \frac{14}{4} = 3.5$$

4 (c) (ii)

$$k = \frac{|st|}{|qr|} \Rightarrow 3.5 = \frac{|st|}{3}$$

$$\therefore |st| = 3 \times 3.5 = 10.5$$

4 (c) (iii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{ (2)}$$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} = \frac{|\text{Area of triangle } pst|}{|\text{Area of triangle } pqr|}$$

$$\Rightarrow 3.5^2 = \frac{|\text{Area of triangle } pst|}{5}$$

$$\Rightarrow |\text{Area of triangle } pst| = 5 \times 3.5^2 = 61.25 \text{ square units}$$

Area of quadrilateral *qstr* = Area of triangle *pst* – Area of triangle *pqr*
= 61.25 – 5 = 56.25 square units

2003

4 (c) (i) Construct a triangle abc in which $|ab| = 10.5$ cm, $|bc| = 5$ cm and $|ac| = 8.5$ cm.

(ii) Choose any point p that is *outside* the triangle and construct the image of abc under the enlargement of scale factor 0.4 and centre p.

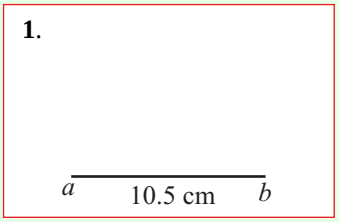
(iii) Given that the area of this image triangle is 3.36 cm^2, calculate the area of the original triangle abc.

SOLUTION

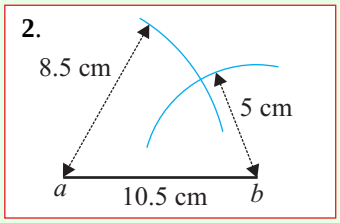
4 (c) (i)

Draw a triangle abc whose sides have lengths of $|ab| = 10.5$ cm, $|ac| = 8.5$ cm and $|bc| = 5$ cm.

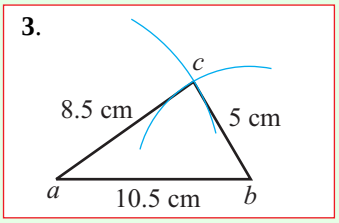
STEP 1: Draw a base using the longest side. Use a ruler to measure out a line segment $[ab]$ of length 10.5 cm.



STEP 2. Take a compass and use a ruler to measure out a length of 8.5 cm. Put the point of the compass at a and draw out an arc of the circle. Do the same for the third side. Using your compass again measure out a length of 5 cm. Put the point of the compass at b and draw out an arc of the circle so that it intersects with the other arc.



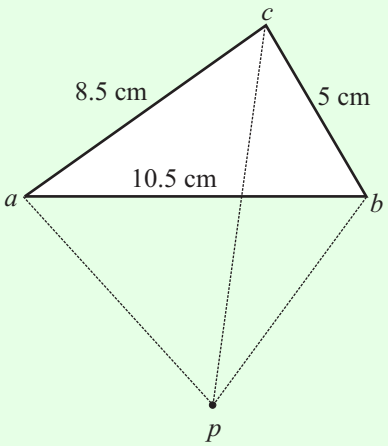
STEP 3. c is the point of intersection of the two arcs. Join a to c and b to c to complete the construction.

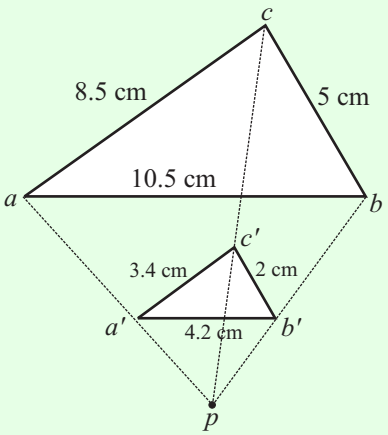


NOTE: Only approximate lengths are shown in the diagrams.

4 (c) (ii)

Mark a point p outside the triangle.
Draw lines from p to each vertex of the triangle.
Mark off the point a' that is 0.4 of the distance $|pa|$.
Do the same for the other 2 points.
Join the points $a'b'c'$ to form the image of the triangle abc.
The lengths of the 3 sides in the image are 0.4 of the lengths of the sides of the object.





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4 (c) (iii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{2}$$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} = \frac{|\text{Area of triangle } a'b'c'|}{|\text{Area of triangle } abc|}$$

$$\Rightarrow 0.4^2 = \frac{3.36}{|\text{Area of triangle } abc|}$$

$$\therefore |\text{Area of triangle } abc| = \frac{3.36}{0.4^2} = 21 \text{ cm}^2$$

2002

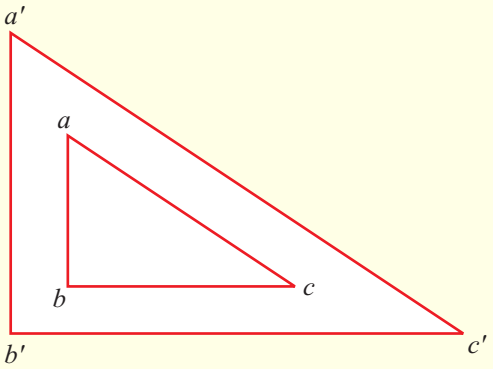
4 (c) The triangle $a'b'c'$ is the image of the triangle abc under an enlargement.

(i) Find, by measurement, the scale factor of the enlargement.

(ii) Copy the diagram and show how to find the centre of the enlargement.

(iii) Units are chosen so that $|bc| = 8$ units.
How many of these units is $|b'c'|$?

(iv) Find the area of triangle abc, given that the area of $a'b'c'$ is 84 square units.



SOLUTION

4 (c) (i)

Measure the length of side $[ab]$ and the length of its image $[a'b']$.

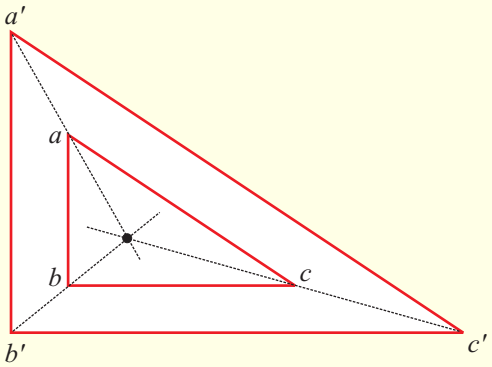
$[ab] = 2$ cm

$[a'b'] = 4$ cm

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \ldots\ldots \text{ (1)}$$

$$\therefore k = \frac{|a'b'|}{|ab|} = \frac{4}{2} = 2$$

4 (c) (ii)



4 (c) (iii)

$$k = \frac{|b'c'|}{|bc|} \Rightarrow 2 = \frac{|b'c'|}{8}$$

$$\therefore |b'c'| = 2 \times 8 = 16 \text{ units}$$

4 (c) (iv)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{ (2)}$$

$$k^2 = \frac{|\text{Area of triangle } a'b'c'|}{|\text{Area of triangle } abc|} \Rightarrow 2^2 = \frac{84}{|\text{Area of triangle } abc|}$$

$$\therefore |\text{Area of triangle } abc| = \frac{84}{2^2} = \frac{84}{4} = 21 \text{ square units}$$

2001

4 (c) (i) Draw a square with sides 7 cm and mark *o*, the point of intersection of the diagonals.

(ii) Draw the image of the square under the enlargement with centre *o* and scale factor $\frac{1}{2}$.

(iii) Calculate the area of the image square.

(iv) Under another enlargement the area of the image of the square with sides 7 cm is 196 cm^2.
What is the scale factor of this englargement?

SOLUTION

4 (c) (i)

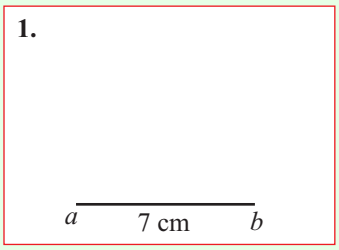
Construct a square *abcd* with side of length 7 cm.

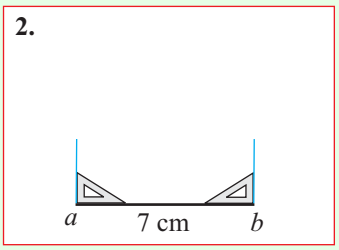
STEP 1. Using a ruler draw a side *ab* of length 7 cm.

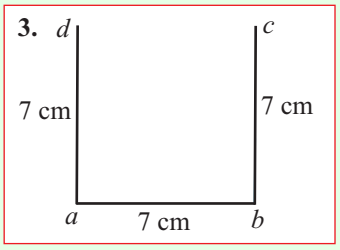
STEP 2. Place the right-angle of a set square on point *a* and draw a light line. Do the same at point *b*.

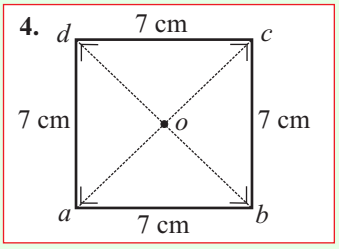
STEP 3. Using the ruler draw a line, *ad*, of length 7 cm from point *a* through the light line. Do the same at point *b* drawing line *bc*.

STEP 4. Complete the square by joining *c* to *d* to form line *cd*. Using your set square, make sure all the angles are right-angled. Using your ruler, make sure each side is of length 7 cm.









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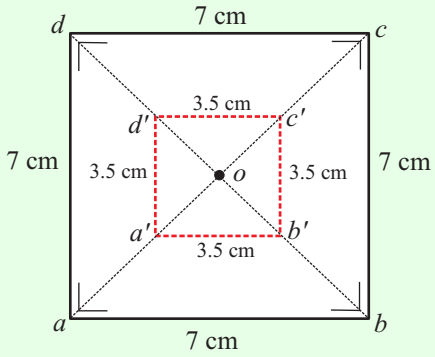
4 (c) (ii)



NOTE: All lengths shown are approximate. When you are doing the question the lengths must be the exact measure.

4 (c) (iii)

$A = 3.5 \times 3.5 = 12.25 \text{ cm}^2$

4 (c) (iv)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{ 2}$$

$|\text{Image area}| = 196 \text{ cm}^2$

$|\text{Object area}| = 7 \times 7 = 49 \text{ cm}^2$

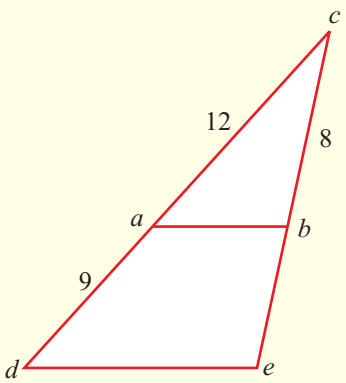
Scale factor k = ?

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \Rightarrow k^2 = \frac{196}{49} = 4$$

$$\therefore k = \sqrt{4} = 2$$

2000

4 (c) The triangle *cde* is the image of the triangle *cab* under an enlargement with centre *c*.
$|ca| = 12$, $|ad| = 9$ and $|cb| = 8$.



(i) Find the scale factor of the enlargement.

(ii) Find $|be|$.

(iii) The area of the triangle *cde* is 98 square units. Find the area of the triangle *cab*.

SOLUTION

4 (c) (i)

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \ldots\ldots \text{ 1}$$

$$k = \frac{|cd|}{|ca|} = \frac{12+9}{12} = \frac{21}{12} = 1.75$$

4 (c) (ii)

$$k = \frac{|ce|}{|cb|} \Rightarrow 1.75 = \frac{|ce|}{8}$$

$$\Rightarrow |ce| = 8 \times 1.75 = 14$$

$$|ce| = |cb| + |be| \Rightarrow 14 = 8 + |be|$$

$$\therefore |be| = 6$$

4 (c) (iii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{ 2}$$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} = \frac{|\text{Area of triangle } cde|}{|\text{Area of triangle } cab|}$$

$$\Rightarrow 1.75^2 = \frac{98}{|\text{Area of triangle } cab|}$$

$$\therefore |\text{Area of triangle } cab| = \frac{98}{1.75^2} = 32 \text{ square units}$$

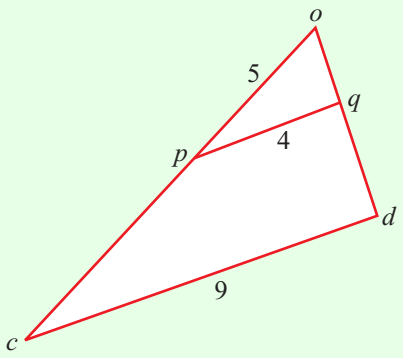
1999

4 (c) The triangle *ocd* is the image of the triangle *opq* under the enlargement, centre *o*, with $|pq| = 4$, $|op| = 5$ and $|cd| = 9$.

(i) Find the scale factor of the enlargement.

(ii) Find $|pc|$.

(iii) The area of the triangle *ocd* is 60.75 square units. Find the area of the triangle *opq*.



SOLUTION

4 (c) (i)

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \ldots\ldots \text{ (1)}$$

$$k = \frac{|cd|}{|pq|} = \frac{9}{4} = 2.25$$

4 (c) (ii)

$$k = \frac{|oc|}{|op|} \Rightarrow 2.25 = \frac{|oc|}{5}$$

$$\Rightarrow |oc| = 5 \times 2.25 = 11.25$$

$$|oc| = |op| + |pc| \Rightarrow 11.25 = 5 + |pc|$$

$$\therefore |pc| = 6.25$$

4 (c) (iii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{ (2)}$$

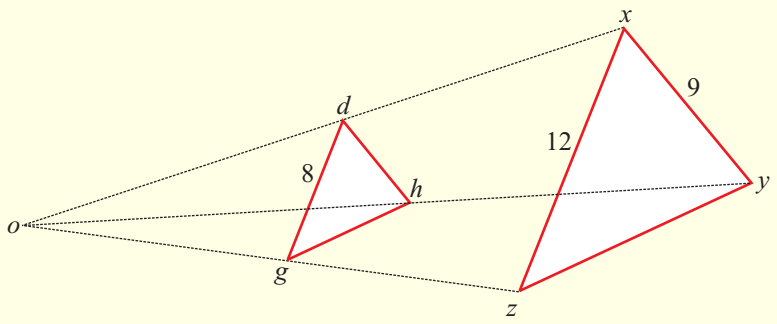
$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} = \frac{|\text{Area of triangle } ocd|}{|\text{Area of triangle } opq|}$$

$$\Rightarrow 2.25^2 = \frac{60.75}{|\text{Area of triangle } opq|}$$

$$\therefore |\text{Area of triangle } opq| = \frac{60.75}{2.25^2} = 12 \text{ square units}$$

1998

4 (c) The triangle xyz is the image of the triangle dgh under the enlargement, centre o, with $|dg| = 8$, $|xz| = 12$ and $|xy| = 9$.



(i) Find the scale factor of the enlargement.

(ii) Find $|dh|$.

(iii) The area of the triangle xyz is 27 square units. Find the area of the triangle dgh.

SOLUTION

4 (c) (i)

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \ldots\ldots \text{ 1}$$

$$k = \frac{|xz|}{|dg|} = \frac{12}{8} = 1.5$$

4 (c) (ii)

$$k = \frac{|xy|}{|dh|} \Rightarrow 1.5 = \frac{9}{|dh|}$$

$$\therefore |dh| = \frac{9}{1.5} = 6$$

4 (c) (iii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{ 2}$$

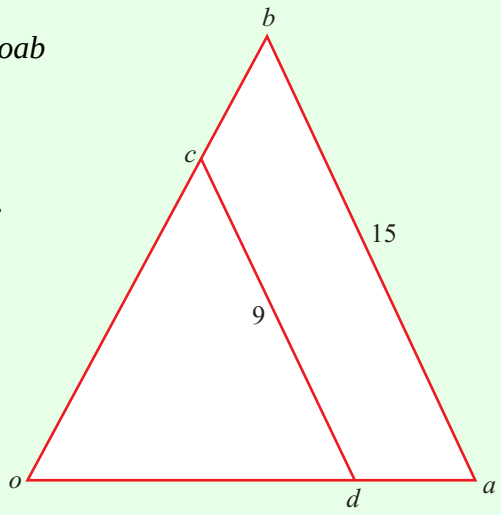
$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} = \frac{|\text{Area of triangle } xyz|}{|\text{Area of triangle } dgh|}$$

$$\Rightarrow 1.5^2 = \frac{27}{|\text{Area of triangle } dgh|}$$

$$\therefore |\text{Area of triangle } dgh| = \frac{27}{1.5^2} = 12 \text{ square units}$$

1997

4 (c) The triangle *odc* is the image of the triangle *oab* under an enlargement, centre *o*.
$|cd| = 9$ and $|ab| = 15$.



(i) Find the scale factor of the enlargement.

(ii) If the area of triangle *oab* is 87.5 square units, find the area of triangle *odc*.

(iii) Write down the area of the region *abcd*.

SOLUTION

4 (c) (i)

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \ldots\ldots \text{ (1)}$$

$$k = \frac{|cd|}{|ba|} = \frac{9}{15} = 0.6$$

4 (c) (ii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{ (2)}$$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} = \frac{|\text{Area of triangle } odc|}{|\text{Area of triangle } oab|}$$

$$\Rightarrow 0.6^2 = \frac{|\text{Area of triangle } odc|}{87.5}$$

$$\therefore |\text{Area of triangle } odc| = 0.6^2 \times 87.5 = 31.5 \text{ square units}$$

4 (c) (iii)

Area of *abcd* = Area of triangle *oab* – Area of triangle *odc* = 87.5 – 31.5 = 56 square units

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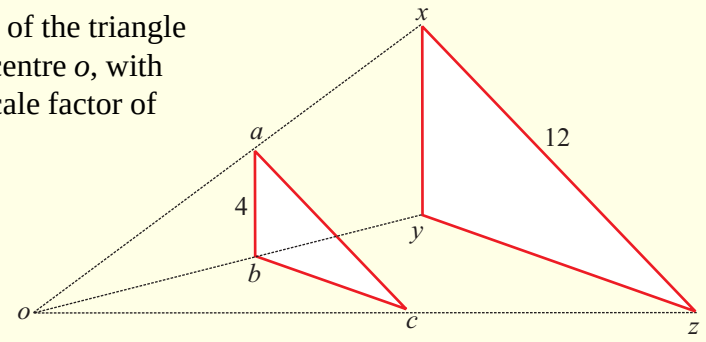
4 (c) The triangle *xyz* is the image of the triangle *abc* under the enlargement, centre *o*, with $|ab| = 4$ and $|xz| = 12$. The scale factor of the enlargement is 1.5.



(i) Find $|xy|$.

(ii) Find $|ac|$.

(iii) If the area of triangle *abc* is 12.2 square units, calculate the area of triangle *xyz*.

SOLUTION

4 (c) (i)

$$k = \frac{|xy|}{|ab|} \Rightarrow 1.5 = \frac{|xy|}{4}$$

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|} \quad \ldots\ldots \text{ (1)}$$

$$\therefore |xy| = 4 \times 1.5 = 6$$

4 (c) (ii)

$$k = \frac{|xz|}{|ac|} \Rightarrow 1.5 = \frac{12}{|ac|}$$

$$\therefore |ac| = \frac{12}{1.5} = 8$$

4 (c) (iii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \text{ (2)}$$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} = \frac{|\text{Area of triangle } xyz|}{|\text{Area of triangle } abc|}$$

$$\Rightarrow 1.5^2 = \frac{|\text{Area of triangle } xyz|}{12.2}$$

$$\therefore |\text{Area of triangle } xyz| = 1.5^2 \times 12.2 = 27.45 \text{ square units}$$