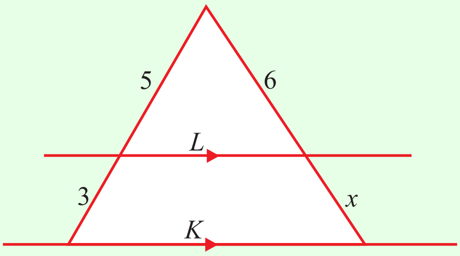
GEOMETRY (Q 4, PAPER 2)

LESSON NO. 2: MORE ABOUT TRIANGLES

2006

4 (a) In the diagram $L \parallel K$.
Find the value of x.



SOLUTION

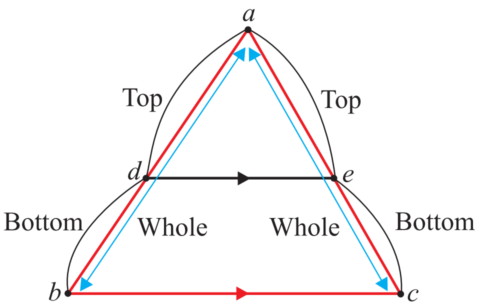
THEOREM 4: A line which is parallel to one side of a triangle and cuts a second side will cut the third side in the same proportion as the second.

You can use any combination of these formulae:

$$\frac{|ad|}{|db|}=\frac{|ae|}{|ec|}=\frac{\text{Top}}{\text{Bottom}} \quad OR$$

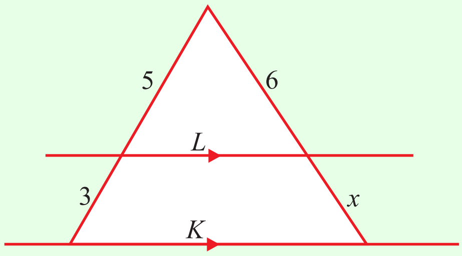
$$\frac{|ab|}{|ad|}=\frac{|ac|}{|ae|}=\frac{\text{Whole}}{\text{Top}} \quad OR$$

$$\frac{|ab|}{|db|}=\frac{|ac|}{|ec|}=\frac{\text{Whole}}{\text{Bottom}}$$



$$\frac{5}{3}=\frac{6}{x} \Rightarrow 5x=18 \text{ [Multiply across by } 3x.]$$

$$\therefore x=\frac{18}{5}=3.6$$

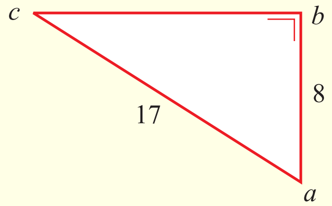


2004

4 (a) In the triangle abc,

$|ab| = 8$, $|ac| = 17$ and $|\angle abc| = 90^{\circ}$.

Find $|bc|$.

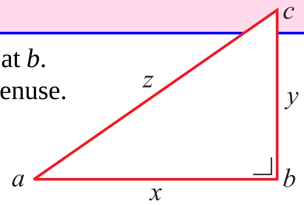


SOLUTION

THEOREM 6: In a right-angled triangle the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

Triangle abc is right-angled with the 90° angle at b.
The side opposite this angle is called the hypotenuse.
This is the formula you use:

$z^2 = x^2 + y^2$



$|ab|^2 + |bc|^2 = |ac|^2$

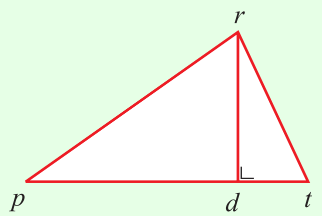
$\Rightarrow 8^2 + |bc|^2 = 17^2 \Rightarrow 64 + |bc|^2 = 289$

$\Rightarrow |bc|^2 = 289 - 64 = 225$

$\therefore |bc| = \sqrt{225} = 15$

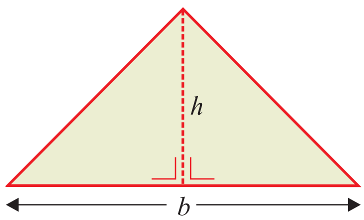
2002

4 (a) The area of the triangle rpt is 30 cm^2.
rd is perpendicular to pt.
Given that $|pt| = 12$cm, calculate $|rd|$.



SOLUTION

4. **NON RIGHT-ANGLED TRIANGLES**



b: Base
h: Height

$\boxed{A = \tfrac{1}{2}bh}$ **4**

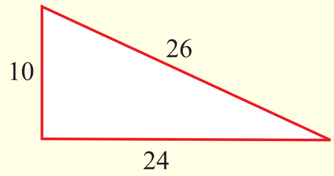
$A = \frac{1}{2}|pt||rd| \Rightarrow 30 = \frac{1}{2}(12) \times |rd|$

$\Rightarrow 30 = 6 \times |rd|$

$\therefore |rd| = 5$ cm

2001

4 (a) Prove that the triangle with sides of lengths 10 units, 24 units and 26 units is right-angled.



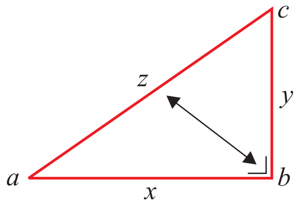
Solution

Theorem 7: (Converse of Pythagoras) If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides then the triangle has a right angle and this is opposite the longest side.

If you can show that $z^2 = x^2 + y^2$

$\Rightarrow \Delta abc$ is a right-angled triangle and

$|\angle abc| = 90^\circ$ is opposite the longest side, z.



$26^2 = 676$

$10^2 + 24^2 = 100 + 576 = 676$

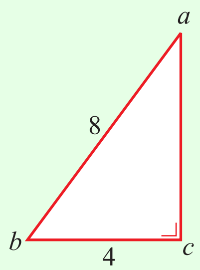
$\therefore 26^2 = 10^2 + 24^2$

Therefore, the triangle is a right-angled triangle.

1999

4 (a) abc is a triangle with $|ab| = 8$, $|bc| = 4$ and $|\angle acb| = 90^\circ$.

Calculate $|ac|$, correct to two places of decimals.



Solution

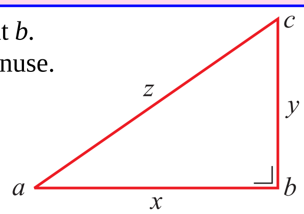
[C] **Pythagoras Theorem**

This theorem only applies to right-angled triangles.

Theorem 6: In a right-angled triangle the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

Triangle abc is right-angled with the 90° angle at b.
The side opposite this angle is called the hypotenuse.
This is the formula you use:

$z^2 = x^2 + y^2$



$|ab|^2 = |ac|^2 + |bc|^2$

$\Rightarrow 8^2 = |ac|^2 + 4^2$

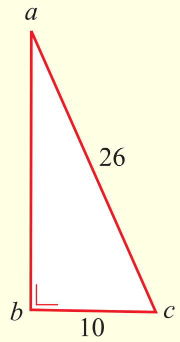
$\Rightarrow 64 = |ac|^2 + 16$

$\Rightarrow |ac|^2 = 48$

$\therefore |ac| = \sqrt{48} = 6.93$

1997

4 (a) Find the area of triangle *abc* if $|\angle abc| = 90^\circ$, $|ac| = 26$ and $|bc| = 10$.



Solution

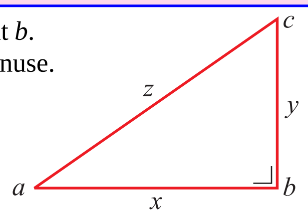
First, find the length of side $|ab|$ using Pythagoras.

[C] **Pythagoras Theorem**
This theorem only applies to right-angled triangles.

Theorem 6: In a right-angled triangle the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

Triangle *abc* is right-angled with the 90° angle at *b*.
The side opposite this angle is called the hypotenuse.
This is the formula you use:

$$z^2 = x^2 + y^2$$

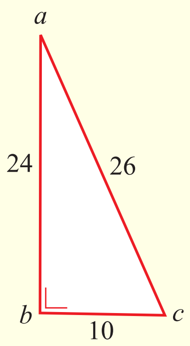


$$|ab|^2 + 10^2 = 26^2$$

$$\Rightarrow |ab|^2 + 100 = 676$$

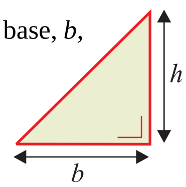
$$\Rightarrow |ab|^2 = 676 - 100 = 576$$

$$\therefore |ab| = \sqrt{576} = 24$$



Area of a right-angled triangle
You can find the area, *A*, by multiplying half the base, *b*, by the perpendicular height, *h*.

$\boxed{A = \frac{1}{2}bh}$ **4**



$$A = \tfrac{1}{2}(10)(24) = 120 \text{ square units}$$