## Geometry (Q 4, Paper 2)

## 2010

4 (a) In the diagram,
$|B C|=|B D|$ and $|\angle A B D|=118^{\circ}$.
(i) Find $x$.
(ii) Find $y$.

(b) Prove that if three parallel lines make intercepts of equal length on a transversal, then they will also make intercepts of equal length on any other transversal.
(c) (i) Draw a square $O A B C$ with side 4 cm and label the vertices.
(ii) Draw the image of the square under the enlargement with centre $O$ and scale factor $2 \cdot 5$.
(iii) Calculate the ratio
area of image square : area of original square.
(iv) Another square, $O P Q R$, is the image of the square $O A B C$ under a different enlargement with centre $O$.
The area of $O P Q R$ is $324 \mathrm{~cm}^{2}$. Calculate the scale factor of this enlargement.

## Solution

4 (a) (i)
$x^{0}=180^{\circ}-118^{\circ}=62^{\circ}$ [Straight angle]

## 4 (a) (ii)

The base angles are equal as the triangle is isosceles. $x^{\circ}+y^{\circ}+y^{\circ}=180^{\circ}$ [3 angles of a triangle add up to $180^{\circ}$ ]
$62^{\circ}+2 y^{0}=180^{\circ}$
$2 y^{\circ}=180^{\circ}-62^{\circ}=118^{\circ}$

$y^{0}=59^{\circ}$

4 (b)
Theorem 3: If three parallel lines make intercepts of equal length on a transversal, then they will make intercepts of equal lengths on any other transversal.

Given: Three parallel lines $a b, c d$ and $e f$ such that $c$ is on $a e$ and $d$ is on $b f$ with $|a c|=|c e|$.


To Prove: $|b d|=|d f|$.
Construction: Draw a line $g h$ through $d$ parallel to $a e$ such that $g$ is on $a b$ and $h$ is on $e f$.


Proof: $a c d g$ is a parallelogram $\Rightarrow|a c|=|g d|=|c e|$
cehd is a parallelogram $\Rightarrow|c e|=|d h|$.
$\therefore|g d|=|d h|$.
Now $\Delta g d b$ and $\Delta f d h$ are congruent (ASA) because:
$|\angle b g d|=|\angle f h d|=A$ [Alternate angles]
$|\angle g d b|=|\angle f d h|=B$ [Vertically opposite angles]
$|g d|=|d h|$ [Already proved]
$\therefore|b d|=|d f|$.

## 4 (c) (i)

Construct a square $O A B C$ with side of length 4 cm .
Step 1. Using a ruler draw a side $O A$ of length 4 cm .
Step 2. Place the right-angle of a set square on point $O$ and draw a light line. Do the same at point $A$.

Ster 3. Using the ruler draw a line, $O C$, of length 4 cm from point $O$ through the light line. Do the same at point $A$ drawing line $A B$.

Step 4. Complete the square by joining $C$ to $B$ to form line $C B$. Using your set square, make sure all the angles are right-angled. Using your ruler, make sure each side is of length 4 cm .


## 4 (c) (ii)

Scale factor $k=2.5$.
The lengths of the image of each side $2.5 \times 4=10 \mathrm{~cm}$


Note: All lengths shown are approximate. When you are doing the question the lengths must be the exact measure.

## 4 (c) (iii)

Area of the image square $=10 \mathrm{~cm} \times 10 \mathrm{~cm}=100 \mathrm{~cm}^{2}$
Area of the original square $=4 \mathrm{~cm} \times 4 \mathrm{~cm}=16 \mathrm{~cm}^{2}$
Area of image square : Area of original square $=100: 16=25: 4$

## 4 (c) (iv)

$k=$ ?
Area of image square $O P Q R=324 \mathrm{~cm}^{2}$
Area of object square $O A B C=16 \mathrm{~cm}^{2}$
$k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid}$ $k^{2}=\frac{324}{16}=\frac{81}{4} \Rightarrow k=\sqrt{\frac{81}{4}}=\frac{9}{2}=4.5$

