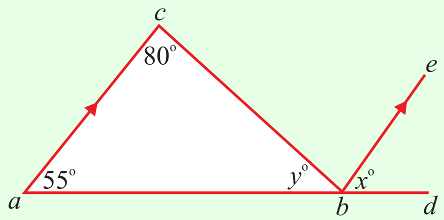
2009

4 (a) In the diagram, *ac* is parallel to *be*,

$|\angle bca| = 80^\circ$ and $|\angle cab| = 55^\circ$.

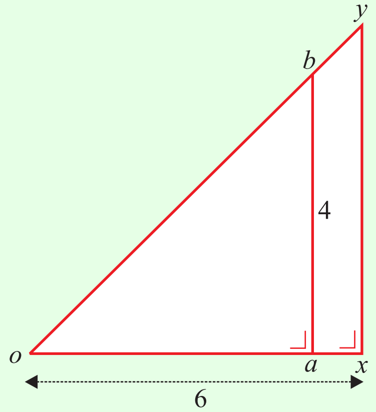
(i) Find x.

(ii) Find y.



(b) Prove that the sum of the lengths of any two sides of a triangle is greater than that of the third side.

(c) The right-angled triangle *oxy* is the image of the triangle *oab* under the enlargement of centre *o* and scale factor 1·2.
$|ab| = 4$ and $|ox| = 6$.



(i) Find $|xy|$.

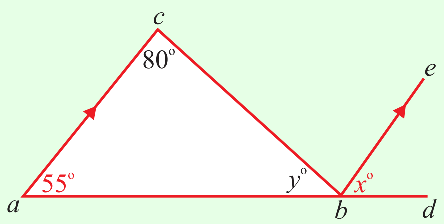
(ii) Find $|oa|$.

(iii) Find the area of the triangle *oab*.

(iv) Find the area of the figure *axyb*.

SOLUTION

4 (a)

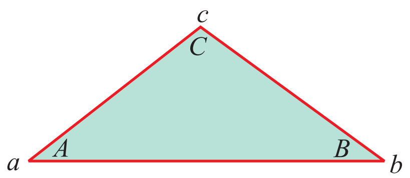


$x^\circ = 55^\circ$ [Corresponding angles]
$y^\circ = 45^\circ$ [3 angles of a triangle add up to 180°]

4 (b)

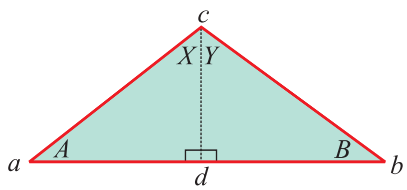
THEOREM 10: The sum of the lengths of any two sides of a triangle is greater than that of the third side.

GIVEN: Δabc



TO PROVE: $|ab| < |ac| + |bc|$

CONSTRUCTION: Draw a perpendicular $[cd]$ onto $[ab]$.



PROOF: $X < 90^\circ \Rightarrow |ad| < |ac|$......**1**

$Y < 90^\circ \Rightarrow |db| < |bc|$......**2**

Adding **1** and **2**: $|ad| + |db| < |ac| + |bc|$

$\therefore |ab| < |ac| + |bc|$.

4 (c) (i)

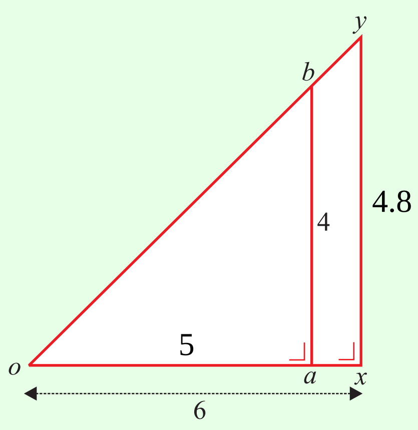
$k = 1.2$

Object length = $|ab| = 4$

Image length = $|xy| = ?$

Scale factor $k = \dfrac{|\text{Image length}|}{|\text{Object length}|}$

$$1.2 = \frac{|xy|}{4} \Rightarrow |xy| = 4 \times 1.2 = 4.8$$



4 (c) (ii)

$k = 1.2$

Object length = $|oa| = ?$

Image length = $|ab| = 6$

$$1.2 = \frac{6}{|oa|} \Rightarrow |oa| = \frac{6}{1.2} = 5$$

4 (c) (iii)

Area of triangle $oab = \frac{1}{2}(5)(4) = 10$ $A = \frac{1}{2}bh$

4 (c) (iv)

$k = 1.2$

Object area = 10

Image area (triangle oxy) = ?

$k^2 = \dfrac{|\text{Image area}|}{|\text{Object area}|}$

$$(1.2)^2 = \frac{|\text{Area of } oxy|}{10} \Rightarrow |\text{Area of } oxy| = 10 \times 1.2^2 = 14.4$$

Area of $axyb$

= Area of oxy – Area of oab

= 14.4 – 10

= 4.4