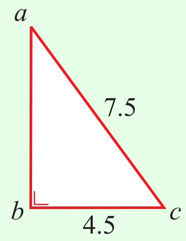
GEOMETRY (Q 4, PAPER 2)

2008

4 (a) In the triangle *abc*,

$|\angle abc| = 90^\circ$, $|bc| = 4{\cdot}5$ and $|ac| = 7{\cdot}5$.

Find $|ab|$.



(b) Prove that the opposite sides of a parallelogram have equal lengths.

(c) (i) Construct an equilateral triangle *pqr* of side 8 cm.

(ii) Construct the image of the triangle *pqr* under the enlargement of scale factor $0{\cdot}75$ and centre *q*.

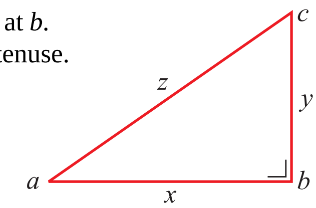
(iii) Given that the area of the triangle *pqr* is $16\sqrt{3}\ \text{cm}^2$, find the area of the image triangle in the form $k\sqrt{3}\ \text{cm}^2$.

SOLUTION

4 (a)

Triangle *abc* is right-angled with the 90° angle at *b*.
The side opposite this angle is called the hypotenuse.
This is the formula you use:

$z^2 = x^2 + y^2$



$|ab|^2 + 4.5^2 = 7.5^2$

$\Rightarrow |ab|^2 = 7.5^2 - 4.5^2 = 36$

$\therefore |ab| = \sqrt{36} = 6$

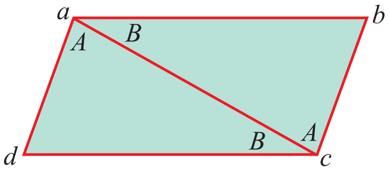
4 (b)

THEOREM 2: The opposite sides of a parallelogram have equal lengths.

GIVEN: Parallelogram *abcd*.

TO PROVE: $|ab| = |dc|$ and $|ad| = |bc|$.

CONSTRUCTION: Join *a* to *c*.



PROOF: Δabc and Δadc are congruent (**ASA**) because:

$|\angle acb| = |\angle dac| = A$ [Alternate angles]

$|\angle bac| = |\angle dca| = B$ [Alternate angles]

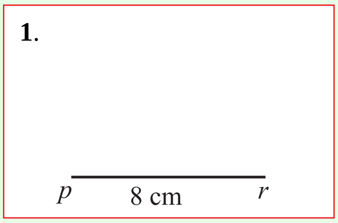
$|ac| = |ac|$ [Common side]

Therefore, $|ab| = |dc|$ and $|ad| = |bc|$.

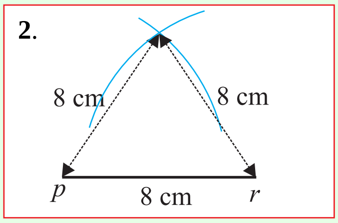
4 (c) (i)

Draw a triangle *abc* whose sides have lengths of $|pr| = 8$ cm, $|pq| = 8$ cm and $|qr| = 8$ cm.

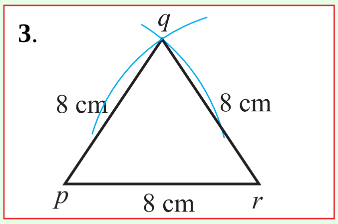
STEP 1: Use a ruler to measure out a line segment [*pr*] of length 8 cm.



STEP 2. Take a compass and use a ruler to measure out a length of 8 cm. Put the point of the compass at *p* and draw out an arc of the circle. Do the same for the third side. Put the point of the compass at *r* and draw out an arc of the circle so that it intersects with the other arc.



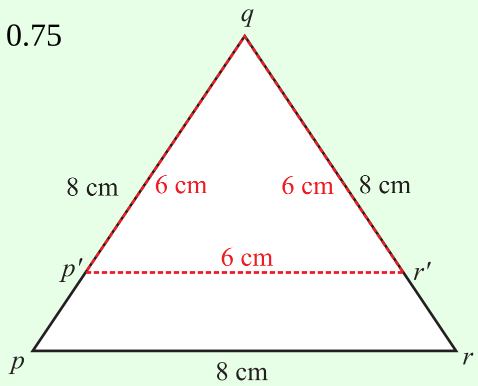
STEP 3. *q* is the point of intersection of the two arcs. Join *p* to *q* and *r* to *q* to complete the construction.



NOTE: Only approximate lengths are shown in the diagrams.

4 (c) (ii)

Multiply the lengths of the lines by the scale factor of 0.75 to find the lengths of their images.



Length of image of $qp = 8 \times 0.75 = 6$ cm

Length of image of $qr = 8 \times 0.75 = 6$ cm

4 (c) (iii)

$$\boxed{k^2 = \frac{|\text{Image area}|}{|\text{Object area}|}} \;\ldots\ldots\; \textbf{2}$$

Object area (triangle pqr) $= 16\sqrt{3}\text{ cm}^2$

Image area (small triangle) = ?

Scale factor $k = 0.75 = \frac{3}{4}$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \Rightarrow \left(\frac{3}{4}\right)^2 = \frac{|\text{Image area}|}{|16\sqrt{3}|}$$

$$\therefore |\text{Image area}| = \left(\frac{3}{4}\right)^2 \times 16\sqrt{3} = 9\sqrt{3}\text{ cm}^2$$