## Geometry (Q 4, Paper 2)

2007
4 (a) In the diagram, two sides of the triangle are produced.
(i) Find $x$.
(ii) Find $y$.

(b) Prove that the products of the lengths of the sides of a triangle by the corresponding altitudes are equal.
(c) The triangle ocd is the image of the triangle oab under an enlargement with centre $o$. $|o a|=4,|a c|=7 \cdot 2$ and $|c d|=7$.

(i) Find the scale factor of the enlargement.
(ii) Find $|a b|$.
(iii) The area of the triangle $o a b$ is 4.5 square units.

Find the area of the triangle ocd.

## Solution

4 (a) (i)
[A] Straight angles: $L$ is a straight line. The angles on $L$ add up to $180^{\circ}$.
$\therefore A+B+C=180^{\circ}$

[E] Angles in a Triangle
Theorem 1: The sum of degree measure of the interior angles of a triangle is $180^{\circ}$.

Stated mathematically: $A+B+C=180^{\circ}$

$A+145^{\circ}=180^{\circ}$ [Straight angle]
$\Rightarrow A=180^{\circ}-145^{\circ}=35^{\circ}$
$45^{\circ}+35^{\circ}+x^{0}=180^{\circ}$ [Theorem 1]
$\Rightarrow 80^{\circ}+x^{\circ}=180^{\circ}$
$\therefore x^{0}=100^{\circ}$


## 4 (a) (ii)

$x^{0}+y^{0}=180^{\circ}$ [Straight angle]
$\Rightarrow 100^{\circ}+y^{\circ}=180^{\circ}$
$\therefore y^{\circ}=80^{\circ}$
4 (b)
Theorem 8: The products of the lengths of the sides of a triangle by the corresponding altitudes are equal.

Given: $\Delta a b c$ with altitudes $[a d]$ and $[c e]$.


To Prove: $|a d||b c|=|a b||c e|$.

Construction: Separate out $\Delta a b d$ and $\Delta e b c$.


Proof: $\Delta a b d$ and $\Delta e b c$ are equiangular because:
$|\angle a d b|=|\angle c e b|=90^{\circ}$
$|\angle a b d|=|\angle e b c|=B$ [Common]
$|\angle d a b|=|\angle e c b|=A$ [Remaining angle]
$\therefore \frac{|a d|}{|c e|}=\frac{|a b|}{|b c|} \Rightarrow|a d||b c|=|a b||c e|$

4 (c) (i)


4 (c) (ii)
$k=\frac{|c d|}{|a b|} \Rightarrow 2.8=\frac{7}{|a b|}$
$\therefore|a b|=\frac{7}{2.8}=2.5$
4 (c) (iii)

$$
\begin{equation*}
k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid} \tag{2}
\end{equation*}
$$

Object area (triangle oab) $=4.5$ square units
Image area $($ triangle $o c d)=$ ?
$k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid} \Rightarrow 2.8^{2}=\frac{\mid \text { Image area } \mid}{|4.5|}$
$\therefore$ Image area $\mid=2.8^{2} \times 4.5=35.28$ square units

