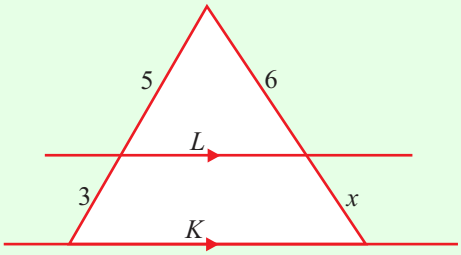
GEOMETRY (Q 4, PAPER 2)

2006

4 (a) In the diagram $L \parallel K$.
Find the value of x.



(b) Prove that if the lengths of two sides of a triangle are unequal, then the degree-measures of the angles opposite to them are unequal, with the greater angle opposite to the longer side.

(c) (i) Construct a triangle abc in which $|ab| = 6.5$ cm, $|bc| = 2.5$ cm and $|ac| = 6$ cm.

(ii) Construct the image of the triangle abc under the enlargement of scale factor 1.8 and centre c.

(iii) Given that the area of triangle abc is 7.5 cm^2, find the area of the image triangle.

SOLUTION

4 (a)

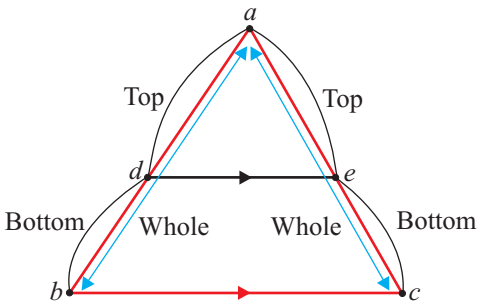
THEOREM 4: A line which is parallel to one side of a triangle and cuts a second side will cut the third side in the same proportion as the second.

You can use any combination of these formulae:

$$\frac{|ad|}{|db|} = \frac{|ae|}{|ec|} = \frac{\text{Top}}{\text{Bottom}} \quad OR$$

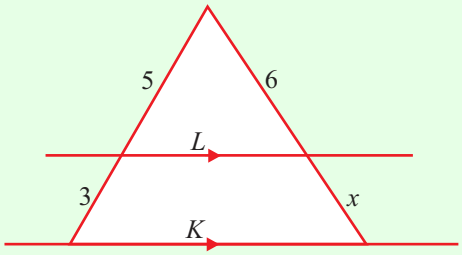
$$\frac{|ab|}{|ad|} = \frac{|ac|}{|ae|} = \frac{\text{Whole}}{\text{Top}} \quad OR$$

$$\frac{|ab|}{|db|} = \frac{|ac|}{|ec|} = \frac{\text{Whole}}{\text{Bottom}}$$



$\frac{5}{3} = \frac{6}{x} \Rightarrow 5x = 18$ [Multiply across by $3x$.]

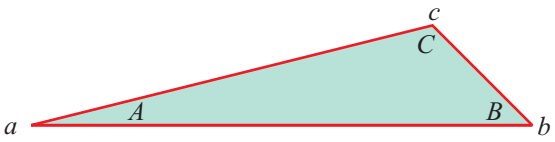
$\therefore x = \frac{18}{5} = 3.6$



4 (b)

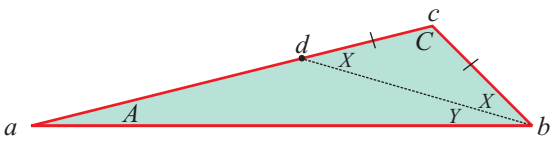
THEOREM 9: If the lengths of a triangle are unequal, then the degree measures of the angles opposite to them are unequal, with the greater angle opposite to the longer side.

GIVEN: Δabc with $|ac| > |bc|$.



TO PROVE: $|\angle abc| > |\angle cab|$.

CONSTRUCTION: Mark a point d on $[ac]$ such that $|cd| = |cb|$.



PROOF: $|dc| = |cb| \Rightarrow |\angle cdb| = |\angle dbc| = X$ [Isosceles triangle]

$B = |\angle abc| = X + Y$

$X = A + Y$ [Exterior Angle]

$\Rightarrow A = X - Y$

$\therefore B > A \Rightarrow |\angle abc| = |\angle cab|$.

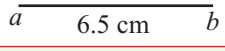
Similarly, this is true for the other sides.

4 (c) (i)

Draw a triangle abc whose sides have lengths of $|ab| = 6.5$ cm, $|ac| = 6$ cm and $|bc| = 2.5$ cm.

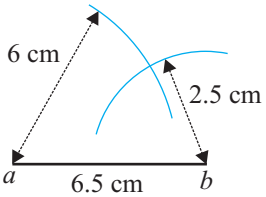
STEP 1: Draw a base using the longest side. Use a ruler to measure out a line segment $[ab]$ of length 6.5 cm.

1.



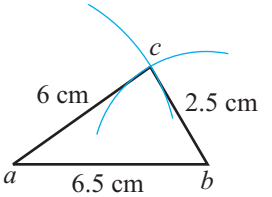
STEP 2. Take a compass and use a ruler to measure out a length of 6 cm. Put the point of the compass at a and draw out an arc of the circle. Do the same for the third side. Using your compass again measure out a length of 2.5 cm. Put the point of the compass at b and draw out an arc of the circle so that it intersects with the other arc.

2.



STEP 3. c is the point of intersection of the two arcs. Join a to c and b to c to complete the construction.

3.



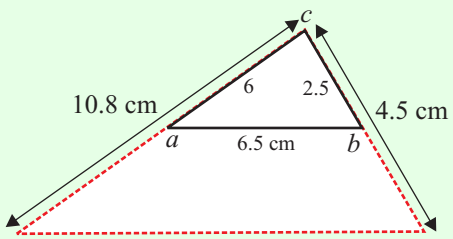
NOTE: Only approximate lengths are shown in the diagrams.

4 (c) (ii)

Multiply the lengths of the lines by the scale factor of 1.8 to find the lengths of their images.

Length of image of $bc = 2.5 \times 1.8 = 4.5$ cm

Length of image of $ac = 6 \times 1.8 = 10.8$ cm



4 (c) (iii)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \ldots\ldots \; 2$$

Object area (triangle *abc*) = 7.5 cm^2

Image area (big triangle) = ?

Scale factor $k = 1.8$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \Rightarrow 1.8^2 = \frac{|\text{Image area}|}{|7.5|}$$

$$\therefore |\text{Image area}| = 1.8^2 \times 7.5 = 24.3 \text{ cm}^2$$