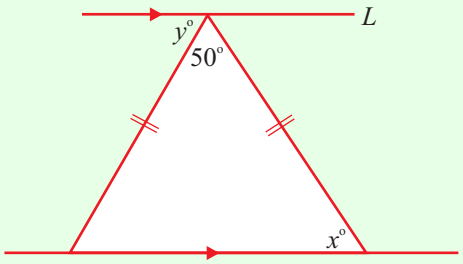
GEOMETRY (Q 4, PAPER 2)

2005

4 (a) In the diagram, the line L is parallel to the base of the isosceles triangle.

(i) Find x.

(ii) Find y.



(b) Prove that a line which is parallel to one side-line of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

(c) (i) Draw a square $opqr$ with sides 8 cm.

(ii) Draw the image of this square under the enlargement with centre o and scale factor 0.25.

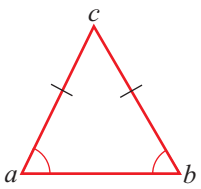
(iii) Calculate the area of this image square.

(iv) Under another enlargement the area of the image of the square $opqr$ is 100 cm^2. What is the scale factor of this enlargement?

SOLUTION

4 (a) (i)

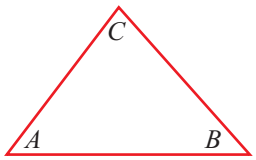
ISOSCELES TRIANGLE: This is a triangle with two equal sides. The angles opposite the equal sides are equal.



[E] **ANGLES IN A TRIANGLE**

THEOREM 1: The sum of degree measure of the interior angles of a triangle is 180°.

Stated mathematically: $A+B+C=180^\circ$

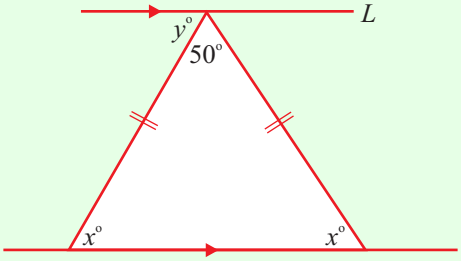


The angles opposite the equal sides are equal.
They are both x°.

$x^\circ + x^\circ + 50^\circ = 180^\circ$ [Theorem 1]

$\Rightarrow 2x^\circ = 130^\circ$

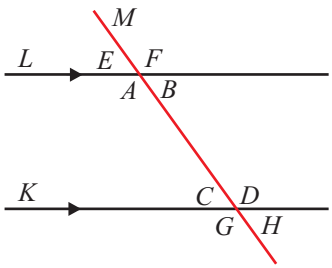
$\therefore x^\circ = 65^\circ$



4 (a) (ii)

[C] **Alternate angles**: K and L are two parallel lines, i.e. $K \parallel L$. A line M cutting these parallel lines is called a transversal. The inside opposite angles are equal and are called alternate angles. Therefore, $A = D$ and $B = C$.

[D] **Corresponding Angles**: On the same diagram, $F = D$, $E = C$, $A = G$ and $B = H$. These are called corresponding angles.



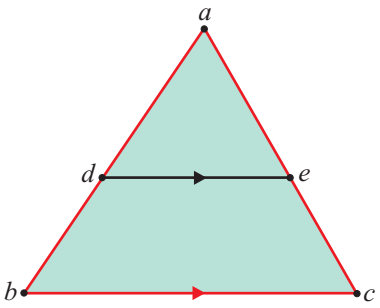
$x^\circ = y^\circ$ [Alternate angles.]

$\therefore y^\circ = 65^\circ$

4 (b)

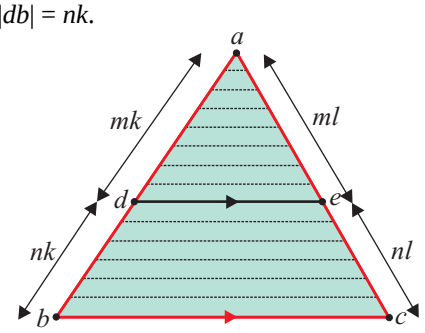
Theorem 4: A line which is parallel to one side of a triangle, and cuts a second side, will cut the third side in the same proportion as the second.

Given: Δabc and line de parallel to bc which cuts $[ab]$ in the ratio $m{:}n$.



To Prove: $\dfrac{|ad|}{|db|} = \dfrac{|ae|}{|ec|} = \dfrac{m}{n}$

Construction: Divide $[ab]$ into m and n parts each of length k so that $|ad| = mk$ and $|db| = nk$.



Proof: According to theorem 3, $[ac]$ is also divided into $m + n$ equal parts each of length l.

$$\frac{|ae|}{|ec|} = \frac{ml}{nl} = \frac{m}{n} = \frac{|ad|}{|db|}.$$

4 (c) (i)

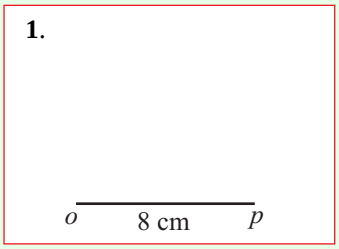
Construct a square *opqr* with side of length 8 cm.

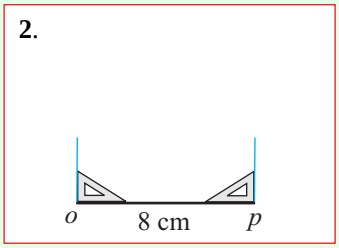
Step 1. Using a ruler draw a side *op* of length 8 cm.

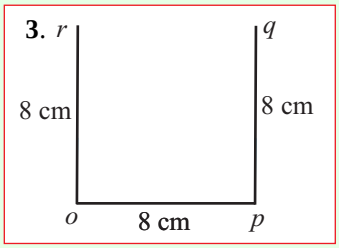
Step 2. Place the right-angle of a set square on point *o* and draw a light line. Do the same at point *p*.

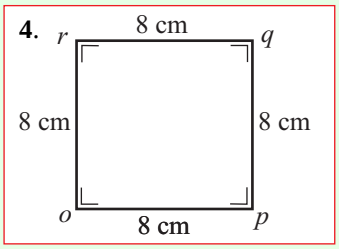
Step 3. Using the ruler draw a line, *or*, of length 8 cm from point *o* through the light line. Do the same at point *p* drawing line *pq*.

Step 4. Complete the square by joining *r* to *q* to form line *rq*. Using your set square, make sure all the angles are right-angled. Using your ruler, make sure each side is of length 8 cm.





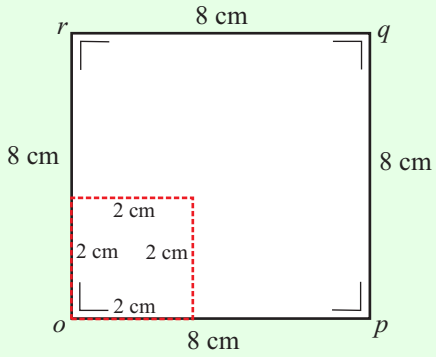




4 (c) (ii)

Scale factor $k = 0.25$

The lengths of the image of each side $8 \times 0.25 = 2$ cm



Note: All lengths shown are approximate. When you are doing the question the lengths must be the exact measure.

4 (c) (iii)

Area of the image square $= 2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$

4 (c) (iv)

Scale factor k = ?

$|\text{Object Area}| = 8 \text{ cm} \times 8 \text{ cm} = 64 \text{ cm}^2$

$|\text{Image Area}| = 100 \text{ cm}^2$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} = \frac{100}{64} = \frac{25}{16}$$

$$\therefore k = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1.25$$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \quad \text{.....} \ 2$$