## Geometry (Q 4, Paper 2)

## 2004

4 (a) In the triangle $a b c$, $|a b|=8,|a c|=17$ and $|\angle a b c|=90^{\circ}$.
Find $|b c|$.

(b) Prove that the opposite sides of a parallelogram have equal lengths.
(c) The triangle $p s t$ is the image of the triangle $p q r$ under an enlargement with centre $p$.

$|p r|=4,|r t|=10$ and $|q r|=3$.
(i) Find the scale factor of the enlargement.
(ii) Find $|s t|$.
(iii) The area of the triangle $p q r$ is 5 square units.

Find the area of the quadrilateral qstr.

## Solution

4 (a)
Theorem 6: In a right-angled triangle the square of the length of the side opposite to the right-angle is equal to the sum of the squares of the lengths of the other two sides.

Triangle $a b c$ is right-angled with the $90^{\circ}$ angle at $b$.
The side opposite this angle is called the hypotenuse.
This is the formula you use:

$$
z^{2}=x^{2}+y^{2}
$$


$|a b|^{2}+|b c|^{2}=|a c|^{2}$
$\Rightarrow 8^{2}+|b c|^{2}=17^{2} \Rightarrow 64+|b c|^{2}=289$
$\Rightarrow|b c|^{2}=289-64=225$
$\therefore|b c|=\sqrt{225}=15$

4 (b)
Theorem 2: The opposite sides of a parallelogram have equal lengths.

Given: Parallelogram abcd.


To Prove: $|a b|=|d c|$ and $|a d|=|b c|$.
Construction: Join $a$ to $c$.


Proof: $\Delta a b c$ and $\Delta a d c$ are congruent (ASA) because:
$|\angle a c b|=|\angle d a c|=A$ [Alternate angles]
$|\angle b a c|=|\angle d c a|=B$ [Alternate angles]
$|a c|=|a c|$ [Common side]
Therefore, $|a b|=|d c|$ and $|a d|=|b c|$.

4 (c) (i)

$$
\text { Scale factor } k=\frac{\mid \text { Image length } \mid}{\mid \text { Object length } \mid}
$$

1
$k=\frac{|p t|}{|p r|}=\frac{4+10}{4}=\frac{14}{4}=3.5$

## 4 (c) (ii)

$k=\frac{|s t|}{|q r|} \Rightarrow 3.5=\frac{|s t|}{3}$
$\therefore|s t|=3 \times 3.5=10.5$

4 (c) (iii)

$$
k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid} \ldots \ldots .
$$

$k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid}=\frac{\mid \text { Area of triangle } p s t \mid}{\mid \text { Area of triangle } p q r \mid}$
$\Rightarrow 3.5^{2}=\frac{\mid \text { Area of triangle } p s t \mid}{5}$
$\Rightarrow \mid$ Area of triangle $p s t \mid=5 \times 3.5^{2}=61.25$ square units
Area of quadrilateral qstr = Area of triangle pst - Area of triangle pqr
$=61.25-5=56.25$ square units

