## Geometry (Q 4, Paper 2)

2003
4 (a) In the diagram, $L, M$ and $N$ are parallel lines. They make intercepts of the indicated lengths on $J$ and $K . a b$ is parallel to $J$.
(i) Write down the length of [ab].
(ii) Write down the length of $[a c]$.

(b) (i) Prove that the sum of the degree-measures of the angles of a triangle is $180^{\circ}$.
(ii) Deduce that the degree-measure of an exterior angle of a triangle is equal to the sum of the degree-measures of the two remote interior angles.
(c) (i) Construct a triangle $a b c$ in which $|a b|=10.5 \mathrm{~cm},|b c|=5 \mathrm{~cm}$ and $|a c|=8.5 \mathrm{~cm}$.
(ii) Choose any point $p$ that is outside the triangle and construct the image of $a b c$ under the enlargement of scale factor 0.4 and centre $p$.
(iii) Given that the area of this image triangle is $3.36 \mathrm{~cm}^{2}$, calculate the area of the original triangle $a b c$.

## Solution

4 (a) (i)
[A] Parallelograms: These are four-sided figures whose opposite sides are parallel.

Theorem 2: The opposite sides of a parallelogram have equal lengths.
$\therefore|a b|=|d c|$ and $|a d|=|b c|$.


The highlighted shape is a parallelogram as $M$ is parallel to $N$ and $a b$ is parallel to $J$.
$|a b|=5$ [Theorem 2]


## 4 (a) (ii)

[B] Three Parallel Lines:
Consider three equally spaced parallel lines $K, L$ and $M$. Two lines called transversals, $X$ and $Y$, cut these parallel lines.

Theorem 3: If three parallel lines make intercepts of equal length on a transversal, then they will make intercepts of equal lengths on any other transversal.
$\therefore|a c|=|c e| \Leftrightarrow|b d|=|d f|$

$|a c|=6$ [Theorem 3]


4 (b) (i) Theorem 1: The sum of the degree measures of the interior angles of a triangle is $180^{\circ}$.

Given: $\Delta a b c$ with angles $A, B$ and $C$.


To Prove: $A+B+C=180^{\circ}$
Construction: Draw a line $L$ through c parallel to the base of the triangle and label angles $X$ and $Y$ as shown.


Proof: $X+Y+C=180^{\circ}$ [Straight angle]
$X=A$ [Alternate angles]
$Y=B$ [Alternate angles]
$\therefore A+B+C=180^{\circ}$

4 (b) (ii)
Deduction 1: The degree measure of the exterior angle of a triangle is equal to the sum of the two remote interior angles.

Given: $\Delta a b c$ with angles $A, B$ and $C$ and exterior angle $D$.


To Prove: $D=A+C$

Proof: $A+B+C=180^{\circ}$ [Theorem 1]
$B+D=180^{\circ}$ [Straight angle]
Therefore, $B+D=A+B+C$
$\therefore D=A+C$.

## 4 (c) (i)

Draw a triangle $a b c$ whose sides have lengths of $|a b|=10.5 \mathrm{~cm},|a c|=8.5 \mathrm{~cm}$ and $|b c|=5 \mathrm{~cm}$.

Step 1: Draw a base using the longest side. Use a ruler to measure out a line segment [ab] of length 10.5 cm .

Step 2. Take a compass and use a ruler to measure out a length of 8.5 cm . Put the point of the compass at $a$ and draw out an arc of the circle. Do the same for the third side. Using your compass again measure out a length of 5 cm . Put the point of the compass at $b$ and draw out an arc of the circle so that it intersects with the other arc.

Ster 3. $c$ is the point of intersection of the two arcs. Join $a$ to $c$ and $b$ to $c$ to complete the construction.


Note: Only approximate lengths are shown in the diagrams.

## 4 (c) (ii)

Mark a point $p$ outside the triangle.
Draw lines from $p$ to each vertex of the triangle.
Mark off the point $a^{\prime}$ that is 0.4 of the distance $|p a|$. Do the same for the other 2 points.
Join the points $a^{\prime} b^{\prime} c^{\prime}$ to form the image of the triangle $a b c$.
The lengths of the 3 sides in the image are 0.4 of the lengths of the sides of the object.


4 (c) (iii)

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k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid}
$$

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$k^{2}=\frac{\mid \text { Image area } \mid}{\mid \text { Object area } \mid}=\frac{\mid \text { Area of triangle } a^{\prime} b^{\prime} c^{\prime} \mid}{\mid \text { Area of triangle } a b c \mid}$
$\Rightarrow 0.4^{2}=\frac{3.36}{\mid \text { Area of triangle } a b c \mid}$
$\therefore \mid$ Area of triangle $a b c \left\lvert\,=\frac{3.36}{0.4^{2}}=21 \mathrm{~cm}^{2}\right.$

